Financial Crises and Systemic Bank Runs in a Dynamic Model of Banking

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Abstract

I present an infinite horizon general equilibrium monetary model of banking with multiple equilibria. In the good equilibrium, all banks are solvent. In the bad equilibrium, many banks are insolvent and subject to runs. The bad equilibrium is also characterized by deflation and a flight to liquidity, and matches other stylized facts of systemic financial crises. The multiplicity of equilibria arises from a strategic complementarity in the decision to fly to liquidity. A sufficiently large monetary injection eliminates the bad equilibrium. However, the size of the monetary injection required to eliminate the equilibrium depends on the tool that the central bank uses to inject money into the economy.

JEL Codes: E44, E52, G21

1 Introduction

A peculiar event of the 2007-2009 US financial crisis was a dramatic increase in the private sector's willingness to hold liquid assets, a "flight to liquidity." The Federal Reserve reacted aggressively at the time, implementing unconventional monetary policies. The flight to liquidity and the interventions of the Fed resulted in an approximately constant price level and a sizable drop in the money multiplier. The Great Depression saw a similar drop in the money multiplier. Friedman and Schwartz (1963) argue that the absence of adequate Federal Reserve intervention at that time generated deep deflation, making what would otherwise have been a modest or deep recession the Great Depression.

During both crises, several financial institutions became insolvent and were subject to runs. More than one-fifth of the commercial banks in the United States suspended operations during the Great Depression (Friedman and Schwartz, 1963). In 2008, the collapse of Lehman Brothers was followed by a "run on repo" and on other institutions not covered by deposit insurance, as Gorton and Metrick (2012a,b) and Schmidt et al. (2016) document.

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Motivated by these events, this paper provides an infinite horizon, general equilibrium model of banking with multiple equilibria. In the good equilibrium, all banks are solvent and there are no runs. In the bad equilibrium, several banks are insolvent and subject to runs at the same time, capturing the systemic nature of financial crises. In the bad equilibrium, distress in the banking sector is associated with deflation, a drop in asset prices and in the money multiplier, and a flight to liquidity (that is, depositors hold more money and fewer deposits at banks, in comparison to the good equilibrium). I use this framework to study the ability of the central bank to eliminate a banking panic using monetary injections.

The two key ingredients that give rise to multiple equilibria are liquidity risk *à la* Diamond and Dybvig (1983) and debt deflation in the spirit of Fisher (1933).[1] While several papers use liquidity risk or debt deflation to model financial crises,[2] this paper explores the interaction of these two elements in giving rise to panic-based bank runs. This interaction produces some new results in the context of panic-based bank runs. Despite modeling liquidity risk as in Diamond and Dybvig, multiple equilibria in my model arise because of a strategic complementarity in the decision to fly to liquidity in anticipation of runs, rather than a strategic complementarity in the ex-post decision to run as in Diamond and Dybvig. This result gives rise to some novel policy implications.

In the policy analysis, I ask whether and how monetary injections eliminate the bad equilibrium. In particular, the central bank can inject money into the economy using two tools: (i) *asset purchases* (i.e., buying assets on the market) or (i) *loans to banks*. The focus on asset purchases and loans to banks is motivated by the tools used by the Federal Reserve to inject money into the economy during the 2008 financial crisis. Using numerical simulations of the model, I show that both tools eliminate the bad equilibrium, provided that the intervention of the central bank is sufficiently large. This result is consistent with the Friedman-Schwartz hypothesis regarding the Great Depression. The key difference between asset purchases and loans to banks is the size of the monetary injection required to eliminate the bad equilibrium. With loans to banks, the size is less than or equal to that of asset purchases.

In the model, households are subject to uninsurable preference shocks that affect the utility of consumption, similarly to Diamond and Dybvig (1983). There is an exogenous supply of two assets in the economy: fiat money and a productive asset (capital). Trading frictions interact with the timing of preference shocks and create a precautionary demand for money, which is required to finance the consumption expenditure. That is, money is a liquid asset that facilitates transactions.[3]

[1] The debt-deflation channel of Fisher (1933) is based on the fact that bank deposits are denominated in nominal terms, and thus deflation increases their real value. Since deposits are liabilities for banks, an increase in their real value creates distress in the financial sector.

[2] See the literature review for more details.

[3] For instance, Krishnamurthy and Vissing-Jorgensen (2012) show that US Treasury securities provide liquidity services.

Capital, however, cannot be used for transactions, but it provides a positive return. More generally, money can be reinterpreted as an asset that provides liquidity services, whereas the return on capital can be reinterpreted as a spread between liquid and illiquid assets.

Banks offer deposits in order to provide money to households on demand. Two elements of banks' balance sheets are crucial. On the liabilities side, deposits are nominal, that is, specified in terms of money. On the asset side, capital held by banks is hit by idiosyncratic shocks (in particular, I consider one-time unanticipated shocks). For an individual bank, a negative shock destroys some of its capital, whereas a positive shock increases its stock of capital. Crucially, banks have some private information about the idiosyncratic shocks; that is, each bank observes its own shock, but it takes time for households to observe the shocks. Thus, households do not know whether their own bank has been hit by a positive or a negative shock. This assumption is motivated by the observation of Gorton (2008), who emphasizes the uncertainty regarding the identities of the financial institutions that incurred significant losses associated with the housing market during the Great Recession.

I analyze three scenarios. The first scenario is a steady-state equilibrium with no shocks to capital. The second scenario is a good equilibrium in which all banks are solvent, despite the one-time unanticipated shocks that hit banks. The third scenario is a bad equilibrium in which some banks become insolvent and are subject to runs when the unanticipated shocks hit banks.

In the steady state and in the good equilibrium, all banks are solvent and deposits overcome the frictions that give rise to the precautionary demand for money. Thus, the logic of the results in these two scenarios is very similar to that in the good equilibrium of Diamond and Dybvig (1983).

In contrast, banks become insolvent and are subject to runs in the bad equilibrium. However, the logic of the results is very different from that in Diamond and Dybvig (1983). A crucial element of the bad equilibrium is a drop in the price of capital, which affects the asset side of banks' balance sheets because banks hold capital, whereas liabilities are constant because deposits are fixed in nominal terms (debt deflation). Note, though, that only banks hit by negative shocks become insolvent, whereas banks hit by positive shocks remain solvent. Crucially, asymmetric information prevents depositors from immediately identifying insolvent banks. As a result, households fly to liquidity; that is, they demand more liquid assets – money – and fewer deposits and capital in anticipation of a possible future run on their own bank. Runs on insolvent banks occur whenever information about banks' balance sheets becomes common knowledge. From a general equilibrium perspective, the increase in the demand for money and the reduction in the demand for capital trigger an increase in the relative price of money to that of capital; that is, the economy experiences deflation (because the price of money is the inverse of the price level) and a drop in the nominal price of capital.[4] Note that this scenario is indeed an equilibrium because the initial drop in the

[4]More precisely, the economy experiences lower-than-anticipated inflation and not necessarily deflation.

price of capital that generates banks' insolvencies is self-fulfilling.

Since deposits are assets that can easily be converted into money, they are part of broad monetary aggregates such as M1. The drop in deposits is therefore equivalent to a drop in M1, and thus in the money multiplier (because of a constant money supply).

The strategic complementarity that gives rise to the bad equilibrium is related to households' decisions to fly to liquidity. If all other households are holding more money because of a fear of runs, a banking panic does happen in equilibrium so that the best response of each single household is to hold more money as well. Different from bank runs in partial equilibrium models, each household does not respond directly to other players' choices. Rather, each household makes its decision based on market prices, even though such prices are affected by other households' choices.

Within the category of bad outcomes, there are actually multiple bad equilibria – more precisely, up to two bad equilibria depending on parameters. The bad equilibria are quantitatively different but qualitatively identical.

The key policy result is that a sufficiently large monetary injection eliminates the bad equilibrium. More precisely, if the central bank fully commits to implementing such a policy in the event of a bad equilibrium, the bad equilibrium is eliminated. Thus, monetary injections are never implemented in equilibrium. However, the size of the injection required to eliminate the bad equilibrium may differ depending on whether the central bank uses asset purchases or loans to banks. If loans to banks have the same seniority as deposits, loans to banks eliminate the bad equilibria with a smaller monetary injection, in comparison to asset purchases. This is because the losses of insolvent banks are borne not only by depositors but also by the central bank. This effect reduces households' incentives to run, weakening or eliminating a key force of the bad equilibrium. If instead loans from the central bank are senior in comparison to deposits, the size of the monetary injection required to eliminate the bad equilibrium is the same as the one required by asset purchases. This is because the central bank does not bear any of the losses of insolvent banks.[5]

1.1 Comparison with the literature

Diamond and Dybvig (1983) formalize the notion of bank runs as panics using multiplicity of equilibria. There are a number of differences between Diamond and Dybvig (1983) and my work. First, there is only one real asset in Diamond and Dybvig (1983), so it is difficult to use their model

[5] In the bank runs literature, two other policies are often analyzed: suspension of convertibility and deposit insurance. The former does not eliminate bad equilibria in this model because it exacerbates the flight to liquidity. However, deposit insurance does eliminate the bad equilibria, but I abstract from deposit insurance in order to focus on monetary injections, motivated by the interventions of the Federal Reserve in the early 1930s and in 2008-2009. Thus, in the model, banks are unregulated institutions that perform maturity transformation without deposit insurance, similar to commercial banks in the 1930s and to the shadow banking system in recent years.

to analyze monetary injections.[6] In contrast, my model has a specific role for money.[7] Second, the model of Diamond and Dybvig (1983) has exogenous asset returns and thus is typically interpreted as a partial equilibrium model of one bank. My analysis is instead based on a general equilibrium model with endogenous returns, and runs are systemic events that involve a fraction of the banking system. Third, the run equilibrium in Diamond and Dybvig (1983) relies on a coordination failure with regard to the decision to run or not run. In my model, the coordination failure is instead based on the decision to fly or not fly to liquidity, which propagates through a general equilibrium effect.

The model of Gertler and Kiyotaki (2015) combines banking panics with the infinite horizon, general equilibrium formulation of business cycle models.[8] Differently from my model, their baseline framework does not include money, information asymmetries, or preference shocks *à la* Diamond and Dybvig (1983), and the channel that gives rise to multiple equilibria is unrelated to debt-deflation and instead similar to the approach of Cole and Kehoe (2000).

Brunnermeier and Sannikov (2011), Carapella (2012), and Cooper and Corbae (2002) present general equilibrium monetary models with banks and, in Carapella (2012) and Cooper and Corbae (2002), with multiple equilibria. However, the role of banks in those models is to intermediate funds between households and firms, whereas I focus on banks as providers of insurance against preference shocks. In Brunnermeier and Sannikov (2011), a shock to financial intermediaries triggers debt deflation, but no bank is insolvent or subject to runs; monetary policies can help banks to recapitalize. In Carapella (2012), multiplicity arises because of a debt-deflation channel similar to my model; policy analysis, however, emphasizes the comparison between monetary injections and deposit insurance, rather than among alternative monetary policy tools as I do. In Cooper and Corbae (2002), multiplicity is related to increasing returns to scale in intermediation. Their analysis focuses on steady states in which banks are either perpetually malfunctioning or well functioning, whereas I analyze a crisis that eventually ends. As a result, in their model, monetary policy eliminates the bad equilibrium by increasing the growth rate of money, whereas in my model monetary policy eliminates a bad equilibrium by increasing the level of money.

From a modeling perspective, the assumptions concerning the structure of trading in my model are very similar to those in Telyukova and Visschers (2013) and are also analogous to those in Bianchi and Bigio (2014), Lagos and Wright (2005), and Lucas (1990).

From an empirical standpoint, runs on the so-called shadow banking system during the 2008

[6] Martin (2006) presents a monetary version of the Diamond and Dybvig model with nominal deposits and shows that an adequate monetary policy stance eliminates bank runs in that framework as well.

[7] Other papers, such as Antinolfi et al. (2001), Allen et al. (2013), Diamond and Rajan (2006), and Robatto (2016), include money in models of banking. However, these papers analyze monetary injections in the context of fundamentals-driven runs.

[8] Other papers in this literature include Angeloni and Faia (2013), Ennis and Keister (2003), Martin et al. (2014), and Mattana and Panetti (2014). A related literature (e.g., Bigio, 2012) incorporates banks in markets with asymmetric information into dynamic general equilibrium models.

financial crisis are discussed, for example, by Brunnermeier (2009), Covitz et al. (2013), Duffie (2010), and Gorton and Metrick (2012a,b), although the debate about their importance is still open (see Krishnamurthy et al., 2014, and Krishnamurthy and Nagel, 2013). Asymmetric information about banks in the Great Recession is discussed by Gorton (2008); Bernanke (2010) and Armantier et al. (2011) emphasize the stigma associated with borrowing from the discount window. Furthermore, runs and asymmetric information about banks are studied and discussed by Friedman and Schwartz (1963) for the Great Depression (a stigma was associated with banks that borrowed from the government-established Reconstruction Finance Corporation), and by Gorton (1988) and Gorton and Mullineaux (1987) for the national banking era (1863-1914).

2 Model

The economy is populated by a unit mass of banks indexed by $b \in \mathbb{B} \equiv [0,1]$ and a double continuum of households indexed by $h \in \mathbb{H} = [0,1] \times [0,1]$. Time is discrete, and each period is divided into two parts, day and night. Capital letters denote quantities and prices during the day, and lowercase letters denote quantities and prices at night. Superscripts h and b refer to household h and bank b.

Appendix A describes an extension to the model that produces a well-defined steady state. Since the extension does not affect the main results of the paper, I postpone it to the Appendix for simplicity.

2.1 Households and banks

Household $h \in \mathbb{H}$ enjoys utility from goods c_t^h consumed at night according to

$$\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \varepsilon_t^h \log c_t^h$$

where ε_t^h is a preference shock realized at the beginning of the night and

$$\varepsilon_t^h = \begin{cases} \bar{\varepsilon} > 0 & \text{(impatient)} \quad \text{with probability } \kappa \\ \underline{\varepsilon} = 0 & \text{(patient)} \quad \text{with probability } 1-\kappa. \end{cases} \tag{1}$$

The preference shock is private information of household h and is i.i.d. over time and across households; the law of large numbers holds for each subset of $\mathbb{H}$ with a continuum of households. I impose the normalization

$$\mathbb{E}(\varepsilon_t) = 1. \tag{2}$$

Therefore, equations (1) and (2) imply $\kappa\bar{\varepsilon} = 1$.

The banking sector is perfectly competitive, and the objective of banks is to maximize profits.

2.2 Assets, trading, and shocks to capital

Assets. There are three assets in the economy: capital, money, and deposits. Capital is in fixed supply $\overline{K}$. The supply of money M_t is chosen by the central bank and $M_t = \overline{M}$ for all t.[9] A deposit issued by bank b is a claim that is redeemable on demand at bank b. The supply of deposits is endogenously determined in equilibrium.

Markets. Trading takes place in a day market and in a night market, as represented in Figure 1.

During the day, there is a Walrasian market in which households and banks trade capital, money, and deposits. The price of money is normalized to one, and Q_t is the price of one unit of capital. Let K_t^h, M_t^h, and D_t^h be the amount of capital, money, and deposits that household $h \in \mathbb{H}$ has after leaving the day market, and similarly K_t^b, M_t^b, and D_t^b for bank $b \in \mathbb{B}$.

After the day market closes, capital produces output with a linear technology $y(K) = ZK$, $0 < Z < \infty$. Total output $y\left(\overline{K}\right) = Z\overline{K}$ is the only consumption good in the economy. There is no depreciation.

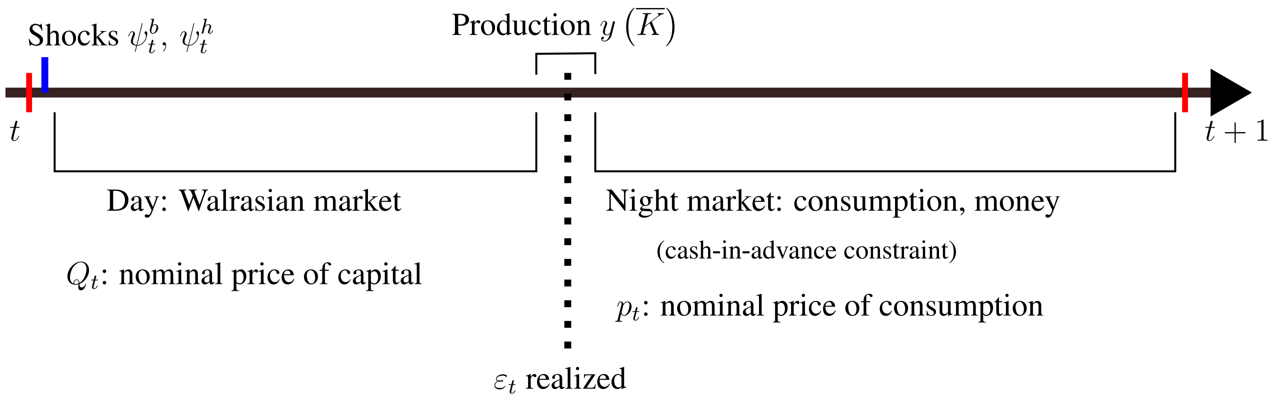
At night, there is another centralized market in which household $h \in \mathbb{H}$ can buy consumption goods c_t^h at price p_t, subject to a cash-in-advance constraint.[10] Capital cannot be traded at night. Let m_t^h and d_t^h be the amount of money and deposits of household $h \in \mathbb{H}$ at the end of the night (to be defined later), and similarly m_t^b and d_t^b for bank $b \in \mathbb{B}$.

State variables and shocks to capital. Each household $h \in \mathbb{H}$ starts the day with a vector of state variables $\boldsymbol{X}_t^h = \left\{\left(K_{t-1}^h, m_{t-1}^h, d_{t-1}^h\right), \psi_t^h\right\}$, where K_{t-1}^h is capital, m_{t-1}^h is money, and d_{t-1}^h are deposits whose values have been determined at $t-1$. The initial stock of capital of household h is $K_{t-1}^h\left(1+\psi_t^h\right)$ where ψ_t^h is an idiosyncratic shock realized at time t with support $\psi_t^h \in \left\{\underline{\psi}, 0, \overline{\psi}\right\}$, $-1 < \underline{\psi} < 0 < \overline{\psi}$. Similarly, $\boldsymbol{X}_t^b = \left\{\left(K_{t-1}^b, m_{t-1}^b, d_{t-1}^b\right), \psi_t^b\right\}$ is the vector of state variables of bank b, and $K_{t-1}^b\left(1+\psi_t^b\right)$ is its initial stock of capital. I will analyze the effects of one-time unanticipated shocks. Formally, $\Pr\left(\psi_t^h = 0 \text{ for all } h \in \mathbb{H}\right) = 1$, and similarly for ψ_t^b. When the one-time unanticipated shocks hit the economy, shocks take the values $\psi_t^h = \underline{\psi}$ (*negative shock*) with probability $\alpha \in (0,1)$ and $\psi_t^h = \overline{\psi}$ (*positive shock*) with probability $1-\alpha$, and similarly for banks. Adding aggregate shocks to capital does not qualitatively affect the main results of the paper.

[9] The assumption $M_t = \overline{M}$ is relaxed in Section 5 in the discussion of monetary policy.

[10] Households cannot consume output produced by their own stock of capital, similarly to standard models with a cash-in-advance constraint such as Lucas and Stokey (1987).

Figure 1: Timing



During the day, the values of ψ_t^h, ψ_t^b are private information of household h and bank b, respectively. However, the values of ψ_t^h, ψ_t^b become common knowledge at night; that is, information acquisition at night is exogenous. The shocks $\{\psi_t^h\}$, $\{\psi_t^b\}$ are idiosyncratic, and the law of large numbers holds both within the banking sector and within the household sector, $\int_{\mathbb{B}} K_{t-1}^b \psi_t^b db = 0$ and $\int_{\mathbb{H}} K_{t-1}^h \psi_t^h dh = 0$.[11]

2.3 Banking

This section describes the interaction between banks and households. I impose a particular demand-deposit contract, rather than deriving it from an explicit contracting problem.

Deposits (day). Household h starts the day with preexisting deposits d_{t-1}^h, and bank b starts with preexisting deposits d_{t-1}^b. These preexisting deposits are a key element of the dynamic nature of the model. While the Walrasian market is open, households and banks choose deposits D_t^h and D_t^b, respectively. Note that the choice of deposits D_t^h taken by household h during the day is a decision regarding rolling over preexisting deposits d_{t-1}^h (fully or partially) or increasing deposits. For instance, if $D_t^h = d_{t-1}^h$, then the dollar value of household h's bank account remains constant. For bank b, the difference $D_t^b - d_{t-1}^b$ is the net issuance of deposits. If $D_t^b > d_{t-1}^b$, bank b increases its deposits and thus receives new resources from households. Otherwise, bank b reduces its amount of preexisting deposits and must pay back some resources to households.[12]

[11] I assume that agents (in particular, banks) cannot insure against this idiosyncratic risk. The results also hold if partial insurance is possible, but not full insurance. The shocks to capital are equivalent, in a three-period model, to heterogeneity in endowment.

[12] To describe precisely the interaction between banks and depositors, I must specify what happens if many preexisting deposits are not rolled over during the day and the bank does not have enough resources to repay them, that is, the bank does not have enough preexisting money m_{t-1}^b and capital $K_{t-1}^b\left(1+\psi_t^b\right)$. If such circumstances occur, the bank is shut down immediately and depositors get pro-rata repayments.

There are two crucial assumptions related to deposits. First, deposits are nominal; that is, they are redeemable for a value specified in terms of money. Second, each household $h \in \mathbb{H}$ can hold deposits D_t^h (at most) at one bank. The latter assumption can be justified by the costs of maintaining banking relationships. Formally, the cost would be zero if household h holds deposits at one bank and infinite if household h holds deposits at two or more banks.[13]

For future reference, let $\mathbb{H}(b) \subset \mathbb{H}$ be the set of depositors of bank $b \in \mathbb{B}$, and let $b(h) \in \mathbb{B}$ be the bank of household $h \in \mathbb{H}$.

Withdrawals (night). At night, households learn the realization of their own preference shock ε_t^h. They then decide to withdraw w_t^h from their own bank subject to a sequential service constraint. Finally, they decide to consume c_t^h.[14]

In the event of large withdrawals from a bank, the bank might not have enough cash to serve all households. Household h can withdraw any amount $0 \leq w_t^h \leq \min\left\{D_t^h, l_t^h\right\}$ where $l_t^h \in \{0, +\infty\}$ is a limit on withdrawals determined by its position in the line. If household h is served when the bank is out of money, then $l_t^h = 0$ and thus $w_t^h = 0$. If household h is served when the bank still has money, then $l_t^h = +\infty$ and $0 \leq w_t^h \leq D_t^h$.

Bank b is *subject to a run* if the limit on withdrawals is $l_t^h = 0$ for some depositor of bank b, that is, for some $h \in \mathbb{H}(b)$. If bank b is subject to a run, the bank is liquidated at $t+1$ while the day market is open.[15] Liquidation works as follows. All assets of the bank are sold on the market, and deposits not withdrawn at night are repaid; if the value of assets is insufficient, depositors are repaid pro-rata.[16]

Because of the cash-in-advance constraint, consumption expenditures $p_t c_t^h$ cannot exceed the sum of money M_t^h chosen during the day and withdrawals w_t^h chosen at night, $p_t c_t^h \leq M_t^h + w_t^h$.

Banks do not make any economic decisions at night. The amount of money withdrawn by depositors of bank b is $w_t^b = \int_{\mathbb{H}(b)} w_t^h dh$. Withdrawals w_t^b are limited by the feasibility constraint $w_t^b \leq M_t^b$ (i.e., money that is distributed at night to depositors cannot exceed the amount M_t^b that bank b held at the end of the day).

Return on deposits. During the day at period t, banks promise to pay a return $1 + R_t^D$ (in $t+1$) on deposits that are not withdrawn that night.[17] However, banks might not have enough resources

[13] The restriction that households can hold deposits only at one bank can be relaxed, but it is crucial that households cannot hold deposits at a large number of banks.

[14] I assume that banks must honor withdrawals as long as they have money in their vaults.

[15] If some bank is liquidated in the day of $t+1$, new banks with no assets and no liabilities enter the market in the day of $t+1$ to keep the measure of banks constant at one.

[16] The case in which the value of assets of the banks is greater than the value of deposits not withdrawn is discussed for completeness in Appendix A, but it is not relevant to the main results of the paper.

[17] The return R_t^D is a market price that is taken as given by both banks and households. The results are unchanged if I allow each bank to post a bank-specific return during the day.

to pay the promised return R_t^D. Define $r_t^b \leq R_t^D$ to be the *actual return on deposits*. Note that the actual return, r_t^b, can be lower than the promised return, R_t^D; if that is the case, the quantity $1+r_t^b$ can be interpreted as the recovery rate. The value of deposits at the end of the night d_t^h for household h is $d_t^h \equiv \left(D_t^h - w_t^h\right)\left(1+r_t^{b(h)}\right)$. For bank $b \in \mathbb{B}$, it is useful to define the value of deposits at the end of the night as $d_t^b \equiv \left(D_t^b - w_t^b\right)\left(1+R_t^D\right)$. That is, the value depends on the promised return R_t^D rather than the actual return r_t^b.

2.4 State of the economy and sunspot

The aggregate state of the economy $\boldsymbol{X}_t$ at the beginning of the day is $\boldsymbol{X}_t = \left\{\Pr_t^B,\, s_t\right\}$, where $\Pr_t^B$ is the probability distribution over the states of banks $\boldsymbol{X}_t^b$ and s_t is a sunspot.[18] The sunspot is an exogenous process that determines equilibrium selection when multiple equilibria exist. The sunspot s_t selects the good equilibrium with probability one, so the bad equilibrium is unanticipated.

For all agents in the model, knowledge of the aggregate state only conveys information about the overall distribution of assets and liabilities of banks. It does not convey any information about the state $\boldsymbol{X}_t^b$ of any particular bank $b \in \mathbb{B}$.

3 Equilibrium

I describe the problem of banks (Section 3.1) and the problem of households (Section 3.2), and then I define the notion of equilibrium (Section 3.4). For future reference, let R_t^K be the nominal return on capital:

$$1 + R_t^K \equiv \frac{Q_{t+1} + Zp_t}{Q_t}. \tag{3}$$

3.1 Bank problem

Given the vector of state variables $\boldsymbol{X}_t^b = \left\{\left(K_{t-1}^b, m_{t-1}^b, d_{t-1}^b\right), \psi_t^b\right\}$ and the price of capital Q_t, the balance sheet of bank b at the beginning of the day is

Assets	*Liabilities*
Value of capital $= K_{t-1}^b\left(1+\psi_t^b\right)Q_t$	Value of deposits $= d_{t-1}^b$
Money $= m_{t-1}^b$	Net worth $= N_t^b$

[18] I will impose restrictions on initial conditions so that the states of banks take finitely many values.

where net worth is the difference between the value of assets and the value of deposits:

$$N_t^b \equiv K_{t-1}^b \left(1+\psi_t^b\right) Q_t + m_{t-1}^b - d_{t-1}^b. \tag{4}$$

If $N_t^b \geq 0$, the bank is solvent (i.e., the value of its assets is larger than deposits d_t^b). If $N_t^b < 0$, the bank is insolvent (i.e., the value of its assets is less than deposits d_t^b). Note that a bank with negative net worth can be active in equilibrium because of asymmetric information about ψ_t^b.

Since bank b takes the price Q_t as given, the net worth N_t^b summarizes the vector of state variables $\boldsymbol{X}_t^b$ for the purpose of understanding the choices of bank b. Given N_t^b and the market return on deposits R_t^D, bank $b \in \mathbb{B}$ chooses deposits D_t^b, money M_t^b, and capital K_t^b in order to solve

$$\max_{D_t^b, M_t^b, K_t^b} \mathbb{E}_{I,\psi}\left[\left(1-I_{t+1}^b\right) N_{t+1}^b + I_{t+1}^b \max\left\{0,\, N_{t+1}^b\right\}\right] \tag{5}$$

subject to the budget constraint

$$K_t^b Q_t + M_t^b \leq D_t^b + N_t^b \tag{6}$$

and non-negativity constraints $D_t^b \geq 0$, $M_t^b \geq 0$, and $K_t^b \geq 0$. The variable I_{t+1}^b is an indicator, $I_{t+1}^b = 1$ if bank b will be liquidated at $t+1$, and $I_{t+1}^b = 0$ if bank b will not be liquidated at $t+1$; the expectation $\mathbb{E}_{I,\psi}$ is taken with respect to I_{t+1}^b and to the shock to capital ψ_{t+1}^b; and N_{t+1}^b is given by equation (4) iterated one period forward. The objective function (8) implies that bank b wants to maximize its net worth in $t+1$, N_{t+1}^b, and takes into account limited liability in the event of liquidation.[19]

In order to complete the description of the law of motion of the state of banks, note that money and deposits at the end of the night at time t, m_t^b and d_t^b, are

$$m_t^b \equiv \left[\left(M_t^b - w_t^b\right) + y\left(K_t^b\right) p_t\right], \quad d_t^b \equiv \left(D_t^b - w_t^b\right)\left(1+R_t^D\right). \tag{7}$$

Restricting the analysis to a scenario that is relevant in equilibrium, the next proposition simplifies problem (5) and provides the solution. The proof is provided in Appendix B.

Proposition 3.1. *Given N_t^b and prices Q_t, $R_t^K, R_t^D \geq 0$, if $r_t^b < R_t^D$ is associated with the liquidation of bank b at time $t+1$, then the objective function (5) simplifies to*

$$\max_{D_t^b, M_t^b, K_t^b} \mathbb{E}_{\psi}\left[\max\left\{0,\, N_{t+1}^b\right\}\right] \tag{8}$$

and the optimal choice of bank b is

[19] The extension in Appendix A motivates (5) by introducing dividends paid by banks, and limited liability arises from bank b paying no dividend if it has negative net worth when it is liquidated.

1. deposits:

$$D_t^b = \begin{cases} 0 & \textit{if } R_t^D > R_t^K \\ \textit{any amount } \geq 0 & \textit{if } R_t^D = R_t^K \\ +\infty & \textit{if } R_t^D < R_t^K; \end{cases}$$

2. money holding $M_t^b = \kappa D_t^b$;

3. capital holding $K_t^b = \frac{N_t^b + D_t^b - M_t^b}{Q_t}$;

provided that the non-negativity constraints $M_t^b \geq 0$ and $K_t^b \geq 0$ are not binding.

To understand the result, consider first a bank with zero net worth, $N_t^b = 0$. The law of large numbers about the preference shocks ε_t^h implies that a fraction κ of depositors withdraw at night to finance consumption expenditures. Thus, banks keep an amount of money $M_t^b = \kappa D_t^b$ that is just enough to finance such withdrawals. The remaining resources $D_t^b - M_t^b = (1-\kappa) D_t^b$ are invested in capital, yielding a net return $(1-\kappa) D_t^b R_t^K$ at $t+1$. Since the bank will have to pay the return $(1-\kappa) D_t^b R_t^D$ on deposits not withdrawn, the profit of the bank is $(1-\kappa) D_t^b \left(R_t^K - R_t^D\right)$. Thus, the bank chooses $D_t^b = 0$ if $R_t^K < R_t^D$ (otherwise it would make negative profit), $D_t^b = +\infty$ if $R_t^K > R_t^D$ (because it can make strictly positive profits on every dollar of deposit), and it is indifferent among any D_t^b if $R_t^K = R_t^D$ (making zero profits).

If a bank has a positive net worth, $N_t^b > 0$, a similar analysis applies. The bank invests a fraction κ of deposits in money and a fraction $1-\kappa$ in capital. The whole net worth N_t^b is invested in capital to maximize the value of net worth tomorrow.

For a bank with negative net worth, $N_t^b < 0$, I explain here only the relevant equilibrium case $R_t^K = R_t^D$. Bank b with negative net worth does not earn profits on deposits if $R_t^K = R_t^D$. Therefore, its net worth at $t+1$ remains negative,[20] and the bank is indifferent among any choice (its payoff will always be zero because of limited liability). Thus, taking the same choices as a solvent bank is (weakly) optimal in the sense that no profitable deviation exists.

Actual return on deposits. The actual return on deposits r_t^b is defined as

$$r_t^b \equiv \min\left\{R_t^D, \widehat{r}_t^b\right\}. \tag{9}$$

The variable $\widehat{r}_t^b$ is the return that can be paid to deposits not withdrawn using proceeds from selling output $ZK_t^b p_t$ and the value of capital $K_t^b Q_{t+1}$ that a bank has at the beginning of $t+1$. Thus, $\widehat{r}_t^b$ solves

$$\mathbb{E}_\psi \left\{K_t^b \left(1+\psi_{t+1}^b\right) Q_{t+1}\right\} + ZK_t^b p_t = \left(D_t^b - w_t^b\right)\left(1+\widehat{r}_t^b\right)$$

[20]Note also that an insolvent bank (i.e., a bank with $N_t^b < 0$) cannot invest 100% of its deposits in money. If $K_t^b = 0$, then $M_t^b = D_t^b + N_t^b$ from the budget constraint (6), and therefore $M_t^b < D_t^b$ because $N_t^b < 0$.

or, using $\psi^b_{t+1} = 0$ with probability one and rearranging

$$1 + \hat{r}^b_t = \frac{K^b_t \left(Q_{t+1} + Zp_t\right)}{D^b_t - w^b_t}. \tag{10}$$

Fraction of depositors served during a run. If all depositors of bank b attempt to withdraw money at night, only a fraction f^b_t of deposits can be withdrawn, defined by

$$f^b_t = \frac{M^b_t}{D^b_t} \tag{11}$$

From the viewpoint of household h that has deposits at bank b, if all depositors of bank b attempt to withdraw their deposits, then household h is able to withdraw with probability f^b_t.

3.2 Household problem

Given the vector of state variables $\boldsymbol{X}^h_t = \left\{\left(K^h_{t-1}, m^h_{t-1}, d^h_{t-1}\right), \psi^h_t\right\}$ of household h and the price of capital Q_t, the nominal wealth A^h_t of household h is

$$A^h_t \equiv K^h_{t-1}\left(1 + \psi^h_t\right) Q_t + m^h_{t-1} + d^h_{t-1}. \tag{12}$$

Household $h \in \mathbb{H}$ is assigned a bank $b\left(h\right) \in \mathbb{B}$. Let $n^h_t = \left\{\varepsilon^h_t, \, r^{b(h)}_t, \, l^h_t\right\} \in \mathcal{N}$ be the vector of variables whose value is learned by household h at night, where

$$\mathcal{N} = \left\{n = \{\varepsilon, \, r, \, l\} \middle|\, \varepsilon \in \{\bar{\varepsilon}, \underline{\varepsilon}\}, \, r \in \mathbb{R}, \, l \in \{0, +\infty\}\right\}.$$

The households' problem can be described in three steps. First, during the day, household h forms beliefs $\Pr^h_t\left(r^{b(h)}_t = r, \, l^h_t = l\right)$ that, combined with the exogenous process for ε^h_t described in (1), imply a probability distribution over $n \in \mathcal{N}$. Second, given the probability distribution over $\mathcal{N}$, household h chooses money M^h_t, deposits D^h_t, and capital K^h_t during the day. Third, at night, household h observes n^h_t and chooses withdrawals $w^h\left(n^h_t\right)$ and consumption $c^h\left(n^h_t\right)$.

Let $V_t\left(A^h_t\right)$ be the value of holding nominal wealth A^h_t. The Bellman equation is

$$V_t\left(A^h_t\right) = \max_{M^h_t, D^h_t, K^h_t} \mathbb{E}_n \left\{ \max_{w^h\left(n^h_t\right), c^h\left(n^h_t\right)} \left[\varepsilon^h_t \log c^h\left(n^h_t\right) + \beta \mathbb{E}_\psi V_{t+1}\left(A^h_{t+1}\left(n^h_t, \psi^h_{t+1}\right)\right)\right]\right\} \tag{13}$$

subject to the budget constraint (14), the limit on withdrawals (15), the cash-in-advance constraint (16), and a non-negativity constraint on money $M^h_t \geq 0$, deposits $D^h_t \geq 0$, and capital $K^h_t \geq 0$:

$$M^h_t + D^h_t + Q_t K^h_t \leq A^h_t \tag{14}$$

$$0 \leq w^h\left(n_t^h\right) \leq \min\left\{D_t^h,\, l_t^h\right\} \tag{15}$$

$$p_t c^h\left(n_t^h\right) \leq M_t^h + w^h\left(n_t^h\right), \tag{16}$$

where the value of wealth $A_{t+1}^h\left(n_t^h, \psi_{t+1}^h\right)$ is

$$A_{t+1}^h\left(n_t^h, \psi_{t+1}^h\right) = \left[K_t^h\left(1+\psi_{t+1}^h\right)\right] Q_{t+1} + d^h\left(n_t\right) + m^h\left(n_t^h\right) \tag{17}$$

and

$$d^h\left(n_t^h\right) \equiv \left[D_t^h - w^h\left(n_t^h\right)\right]\left(1 + r_t^{b(h)}\right) \tag{18}$$

$$m^h\left(n_t^h\right) \equiv \left[M_t^h + w^h\left(n_t^h\right) - p_t c^h\left(n_t^h\right)\right] + p_t\, Z K_t^h. \tag{19}$$

The term $d^h\left(n_t^h\right)$ is the sum of deposits not withdrawn, $D_t^h - w^h\left(n_t^h\right)$, and the actual return $r_t^{b(h)}$ paid by bank $b\left(h\right)$. The term $m^h\left(n_t^h\right)$ is money at the end of the night, which is the sum of the unspent money at night (i.e., money held during the day M_t^h plus withdrawals $w^h\left(n_t^h\right)$ minus the consumption expenditure $c^h\left(n_t^h\right) p_t$) plus the proceeds from selling output ZK_t^h at night at price p_t. The expectation $\mathbb{E}_n$ is taken with respect to the beliefs over $n \in \mathcal{N}$, and the expectation $\mathbb{E}_\psi$ is taken with respect to the shock to capital ψ_{t+1}^h.

If $R_t^D = R_t^K$ and household h has belief $r_t^{b(h)} = R_t^D$ with probability one (which is the case in the good equilibrium), the return on capital is the same as the return on deposits. Thus, households are indifferent between directly investing a fraction of their wealth in capital or depositing more and letting banks buy capital on their behalf. To simplify the derivation, I impose Assumption 3.2.

Assumption 3.2. *If household $h \in \mathbb{H}$ is indifferent among several choices of D_t^h, the household selects the smallest D_t^h that maximizes its utility.*

Assumption 3.2 implies that households use banks only to insure against liquidity risk – that is, they plan to withdraw all their own deposits at night if they are impatient – and invest directly in capital all the wealth they want to carry to $t+1$. The assumption is irrelevant for the bad equilibrium because the optimal D_t^h is unique, whereas it matters in the good equilibrium. However, in the good equilibrium, Assumption 3.2 has no effects on prices.

Proposition 3.3 states the solution to problem (13), focusing on the relevant case $R_t^D = R_t^K$. The proof is provided in Appendix B.

Proposition 3.3. *Given beliefs $Pr_t^h\left(\cdot\right)$ and prices Q_t, R_t^K, and $R_t^D = R_t^K$:*

- *(day) all households invest the same fraction of their initial wealth A_t^h in money (i.e., $M_t^h/A_t^h = M_t^{h'}/A_t^{h'}$ for all $h, h' \in \mathbb{H}$), and similarly for deposits and capital;*

- *(night) withdrawals are*

$$w^h\left(n_t^h\right)=\begin{cases}D_t^h & \text{if } \varepsilon_t^h=\bar{\varepsilon},\quad r_t^{b(h)}\in\mathbb{R},\quad \text{and } l_t^h=+\infty\\ D_t^h & \text{if } \varepsilon_t^h=0,\quad r_t^{b(h)}<0,\quad \text{and } l_t^h=+\infty\\ 0 & \text{otherwise}\end{cases}$$

and consumption is

$$c^h\left(n_t^h\right)=\begin{cases}0 & \text{if } \varepsilon_t^h=0\\ \frac{M_t^h+w^h\left(n_t^h\right)}{p_t} & \text{if } \varepsilon_t^h=\bar{\varepsilon}.\end{cases}$$

Since the felicity from consumption is log, I guess and verify that households' choices during the day are proportional to initial wealth A_t^h. At night, an impatient household ($\varepsilon_t^h=\bar{\varepsilon}$) withdraws deposits if unconstrained (i.e., if $l_t^h=+\infty$) and uses money, M_t^h, and withdrawals, $w^h\left(n_t^h\right)$, to finance its consumption expenditures. If the household is patient ($\varepsilon_t^h=0$), its choice of consumption is zero, but the household is nonetheless willing to withdraw if the actual return on deposits is negative ($r_t^{b(h)}<0$). In this crucial case, the nominal return on money is zero, thus higher than the nominal return on deposits not withdrawn. The household runs on the bank and withdraws all the available deposits D_t^h if the bank still has money while household h is served ($l_t^h=+\infty$). If instead the bank has no money ($l_t^h=0$), the household is stuck with zero withdrawals and receives a negative return on deposits.

Since M_t^h, D_t^h, and K_t^h are proportional to initial wealth A_t^h, Corollary 3.4 holds.

Corollary 3.4. *The choices $\left\{M_t^h,\, D_t^h,\, K_t^h\right\}_{h\in\mathbb{H}}$ of the household sector can be described by a representative household with initial wealth $\overline{A}_t\equiv\int_{\mathbb{H}}A_t^h dh$.*

Consequently, the shocks ψ_t^h to the capital owned by the household sector are irrelevant from an equilibrium perspective because they simply modify the distribution of wealth, but they do not influence the total value of wealth $\overline{A}_t$. It is crucial, however, that the idiosyncratic shocks to capital hit the balance sheet of banks, creating heterogeneity in the banking sector.

3.3 Market-clearing conditions

The market-clearing conditions for capital, money, and deposits traded during the day are

$$\int_{\mathbb{B}}K_t^b db+\int_{\mathbb{H}}K_t^h dh=\overline{K},\qquad \int_{\mathbb{B}}M_t^b db+\int_{\mathbb{H}}M_t^h dh=\overline{M},\qquad \int_{\mathbb{B}}D_t^b db=\int_{\mathbb{H}}D_t^h dh \tag{20}$$

whereas the market-clearing condition for the goods market at night is

$$\int_{\mathbb{H}} c_t^h dh = Z\overline{K}. \tag{21}$$

3.4 Equilibrium definition

The concept of equilibrium combines a standard definition with the requirement that households' beliefs over $r_t^{b(h)}$ (i.e., the actual return on deposits) and l_t^h (i.e., the limit on withdrawal determined by the position in line) are rational. To formalize this notion of equilibrium, let

$$\text{Pr}_t^{(r,l)}\left(r_t^{b(h)} = r,\ l_t^h = l\right),\ r \in \mathbb{R} \text{ and } l \in \{0, +\infty\} \tag{22}$$

be the probability distribution over the actual return on deposits $r_t^{b(h)}$ and the limit on withdrawal l_t^h for household $h \in \mathbb{H}$. The requirement of rational beliefs can thus be stated as an equivalence between households' beliefs Pr_t^h and the probability distribution (22).

The probability distribution (22) can be derived by combining the state of the economy $\boldsymbol{X}_t$ (in particular, Pr_t^B, see Section 2.4) and prices, as follows. First, Pr_t^B provides information about the state variables of banks $\{\boldsymbol{X}_t^b\}_{b\in\mathbb{B}}$; combining $\boldsymbol{X}_t^b$ and the price of capital Q_t, we can use equation (4) to compute the net worth N_t^b of each bank. Second, the net worth N_t^b determines the choices of money M_t^b, deposits D_t^b, and capital K_t^b (see Proposition 3.1). And third, $\{D_t^b, M_t^b, K_t^b\}_{b\in\mathbb{B}}$ determine $\{r_t^b\}_{b\in\mathbb{B}}$ and $\{f_t^b\}_{b\in\mathbb{B}}$ (see equations (9) and (11)), and thus $\{l_t^h\}_{b\in\mathbb{B}}$.[21]

Because of the asymmetric information about banks, there is an issue of whether the equilibrium is pooling or separating. Since I force R_t^D to be equalized across all banks, I am imposing a pooling equilibrium in the banking market. However, the results are unchanged if I allow each bank b to post a bank-specific promised return on deposits. In this case, the equilibrium that arises is still a pooling one because bad banks want to imitate good banks in order to survive as long as possible.

The next definition formalizes the equilibrium concept.

Definition 3.5. *Given the initial state of the economy $\boldsymbol{X}_t$, an equilibrium is a collection of*

- *prices Q_t and p_t and return on capital R_t^K and on deposits R_t^D;*
- *household beliefs $Pr_t^h(\cdot)$, for all $h \in \mathbb{H}$;*
- *household choices $\left\{M_t^h, D_t^h, K_t^h, \{w^h(n_t^h), c^h(n_t^h)\}_{n_t^h\in\mathcal{N}}\right\}$ for all $h \in \mathbb{H}$;*
- *bank choices $\{D_t^b, M_t^b, K_t^b\}$ for all $b \in \mathbb{B}$;*
- *limits on withdrawals $l_t^h \in \{0, +\infty\}$ for all $h \in \mathbb{H}$;*

[21] Recall from Section 3.1 that a household is served ($l_t^h = +\infty$) with probability $f_t^{b(h)}$ if all depositors run on bank b.

- *actual return on deposits r_t^b and fraction of depositors served in the event of a run f_t^b, for all $b \in \mathbb{B}$;*

such that

- *(optimality) households solve problem (13) and banks solve problem (8);*
- *(consistency of withdrawals) if withdrawals $w^h\left(n_t^h\right)$ are constrained to be zero for some household h, then the constraint $w_t^b \leq M_t^b$ must bind;*[22]
- *(rational expectations) households' beliefs are rational, that is, for all $h \in \mathbb{H}$,*

$$\boldsymbol{Pr}_t^h\left(r_t^{b(h)} = r,\, l_t^h = l\right) = \boldsymbol{Pr}_t^{(r,l)}\left(r_t^{b(h)} = r,\, l_t^h = l\right), \quad r \in \mathbb{R} \textit{ and } l \in \{0, +\infty\}$$

- *(market clearing) the market clearing conditions hold.*

I focus on symmetric equilibria in which banks with the same net worth make the same choices, in particular for deposits ($N_t^b = N_t^{b'} \Rightarrow D_t^b = D_t^{b'}$ for $b, b' \in \mathbb{B}$).

4 Results

I first describe the steady state with no shocks to capital. Then, starting from the economy in steady state, I consider the effects of one-time, unanticipated idiosyncratic shocks to capital at time t, $\psi_t^b, \psi_t^h \in \left\{\underline{\psi}, \overline{\psi}\right\}$. At time t, multiple equilibria exist: a good equilibrium in which prices and aggregate quantities are the same as in the steady state, and up to two bad equilibria described in Section 4.2. If the economy experiences a crisis at time t (bad equilibrium), the crisis lasts one period, and the economy is in steady state from $t+1$ onward.

I impose two restrictions on initial conditions. First, all banks are initially alike; second, banks' holdings of capital and money are large enough to guarantee that all banks, including those hit by the negative shock $\underline{\psi}$, are solvent in the good equilibrium.[23]

Assumption 4.1. *At time t, the vector of state variables $\boldsymbol{X}_t^b$ is the same for all banks, that is, $\boldsymbol{X}_t^b = \boldsymbol{X}_t^{b'}$ for all $b, b' \in \mathbb{B}$, and satisfy*

$$K_{t-1}^b\left(1+\underline{\psi}\right)\left[\frac{\beta}{1-\beta} + \left(\frac{1}{\kappa} - 1\right)\right]\frac{\overline{M}}{\overline{K}} + m_{t-1}^b - d_{t-1}^b \geq 0. \tag{23}$$

When (23) holds with equality, banks hit by $\underline{\psi}$ are solvent in the good equilibrium, but their net worth is zero. In the numerical examples used to study the bad equilibrium and the policy

[22] That is, if household h is unable to withdraw at night, the bank of household h must have used all its money to pay withdrawals to other depositors.

[23] Relaxing Assumption 4.1 allows the analysis of a scenario in which runs and insolvencies are driven by fundamentals instead of panics (i.e., a scenario in which there are runs and insolvencies in all equilibria).

analysis, I focus on the parameterization of the model in which (23) holds with equality. If I study economies in which (23) holds with inequality but the left-hand side of (23) is not too large, all the results are qualitatively unchanged. If instead the left-hand side of (23) is sufficiently large, banks have a very large net worth buffer and thus do not become insolvent in the event of a panic. As a result, the bad equilibrium does not exist in this case.

4.1 Steady state and good equilibrium

Steady state. In steady state, prices are constant ($Q_t = Q^*$, $p_t = p^*$, and $R_t^K = R^*$). Because of Assumption 4.1, all banks are identical and solvent. The return on deposits is equal to the return on capital ($R_t^D = R_t^K = R^*$), all banks pay the promised return on deposits that are not withdrawn ($r_t^b = R_t^D = R^*$), and there are no runs ($l_t^h = +\infty$ for all h). Therefore, banks pool the liquidity risk of households, insuring them against preference shocks. The representative household holds deposits D^* and no money ($M_t^h = 0$) because the well-functioning banking system offsets the precautionary demand for money. Withdrawals at night are used to finance the consumption expenditure. Appendix A.3 presents the complete characterization of Q^*, p^*, R^*, and D^* in closed form, as functions of the parameters.[24]

Good equilibrium. If idiosyncratic shocks to capital $\psi_t^b, \psi_t^h \in \{\underline{\psi}, \overline{\psi}\}$ hit the economy, Assumption 4.1 guarantees that a good equilibrium exists. The idiosyncratic shocks imply a redistribution of capital within the banking sector and the household sector, but prices and aggregate quantities in the good equilibrium are the same as in the steady state. Intuitively, since the shocks are idiosyncratic, they have no effects on aggregate variables because all banks remain solvent. The characterization of the good equilibrium is provided in Appendix A.3.

4.2 Bad equilibria

When unanticipated shocks to capital $\underline{\psi}$ and $\overline{\psi}$ hit the economy, the good equilibrium is not the unique one for a large subset of the parameter space. Depending on parameters, there can be up to two bad equilibria in which banks hit by $\underline{\psi}$ become insolvent and are subject to runs.[25] For simplicity, the crisis lasts one period, and then the economy reverts to a new steady state in $t + 1$.

[24]The existence of a well-defined steady state requires an extension described in Appendix A.

[25]When I solve the model with several values of the parameters, I can find at most two equilibria in which only banks hit by $\underline{\psi}$ become insolvent and are subject to runs. Moreover, an additional bad equilibrium exists in which all banks are insolvent ($N_t^b < 0$ for all b, rather than just for banks hit by $\underline{\psi}$) under some conditions on the state of the economy. However, the existence of this equilibrium is sensitive to some timing assumptions related to how new banks enter the banking market. See Appendix C for more details.

This section describes the qualitative features of a bad equilibrium, and the next section presents a numerical example.

The channel that gives rise to bad equilibria. Since all banks are alike in $t-1$, and shocks to capital can take only two values, $\psi_t^b \in \{\underline{\psi}, \overline{\psi}\}$, in equilibrium there are two groups of banks. I will use $N_t(\underline{\psi})$ and $r_t(\underline{\psi})$ to denote the net worth and the actual return on deposits of bank b hit by shock $\psi_t^b = \underline{\psi}$, and similarly $N_t(\overline{\psi})$ and $r_t(\overline{\psi})$ for a bank hit by $\overline{\psi}$.

A bad equilibrium at time t is characterized by four features.

1. The price level is $p_t < p^*$, and the nominal price of capital is $Q_t < Q^*$; that is, the economy experiences deflation and a drop in (nominal) asset prices.
2. Banks hit by the bad idiosyncratic shock $\underline{\psi} < 0$ are insolvent, $N_t(\underline{\psi}) < 0$; banks hit by $\overline{\psi}$ are solvent, $N_t(\overline{\psi}) > 0$.
3. Insolvent banks pay a negative actual return on deposits, $r_t(\underline{\psi}) < 0 < R_t^D$, and are subject to runs at night; solvent banks pay the promised return $R_t^D > 0$ and are not subject to runs.
4. The representative household holds deposits $D_t^h < D^*$ and money $M_t^h > 0$ (flight to liquidity).

The drop in the nominal price of capital Q_t (Item 1) triggers the insolvency of the weakest banks in the economy, that is, of banks hit by $\underline{\psi}$ (Item 2). Recall, from equation (4), that the net worth of a bank hit by $\underline{\psi}$ is $N_t(\underline{\psi}) = K_{t-1}^b(1+\underline{\psi})Q_t + m_{t-1}^b - d_{t-1}^b$ where d_{t-1}^b is the value of preexisting deposits, determined at $t-1$. Therefore, a large enough drop in Q_t implies that net worth $N_t(\underline{\psi})$ is negative. Insolvent banks pay a low actual return on deposits, $r_t(\underline{\psi}) < R_t^D$ (Item 3) because they are insolvent and thus do not have enough resources. To clarify this point, note that the insolvency at the beginning of time t ($N_t(\underline{\psi}) < 0$) and the fact that banks make zero profits on deposits by operating at time t imply that these banks will be insolvent at $t+1$ as well ($N_{t+1}(\underline{\psi}) < 0$); as a result, these banks will be unable to pay the promised return on deposits at the beginning of $t+1$. In particular, the actual return on deposits is negative, $r_t(\underline{\psi}) < 0$, whereas the return from withdrawing and holding money is zero. Therefore, running at night is the optimal choice of depositors of such banks. The flight to liquidity during the day (Item 4) is a result of the anticipation of the runs that will occur at night (Item 3). During the day, each household is aware that it might end up at the end of the line in a run at night and thus unable to withdraw. In response to this belief, households hold more money ($M_t^h > 0$) and fewer deposits at banks ($D_t^h < D^*$) during the day, and they also decrease their demand for capital; that is, they fly to liquidity in order to partially self-insure against the need to finance their consumption expenditure at night.

The scenario described in Items 1-4 is an equilibrium because there is a feedback from the flight to liquidity (Item 4) to the drop in prices (Item 1). This is because the flight to liquidity and away from less liquid assets (i.e., away from deposits and capital) reduces the demand for capital, which in turn produces a drop in the nominal price of capital, $Q_t < Q^*$.

In addition, the flight to liquidity gives rise to deflation. During the day, all households hold a positive amount of money $M_t^h > 0$, but at night some of these households have a zero preference shock ($\varepsilon_t^h = 0$) and choose not to consume. The money held by households with $\varepsilon_t^h = 0$ is thus unspent and "stored under the mattresses," so less money is used for transactions in the economy.[26] In order to understand the implications of the unspent money, multiply both sides of the goods market-clearing condition in (21) by p_t:

$$\underbrace{p_t \int_{\mathbb{H}} c_t^h dh}_{\text{consumption expenditure}} = p_t Z\overline{K}.$$

In the good equilibrium, the left-hand side is equal to $\overline{M}$ because all the money is spent, so $p_t = p^* \equiv \overline{M}/\left(Z\overline{K}\right)$. In the bad equilibrium, the left-hand side is less than $\overline{M}$ because some money in the economy is unspent; therefore, $p_t < p^*$.[27]

Note that, since the shock to banks ψ_t^b takes only two values, there are only two groups of banks. As a result, the measure of insolvent banks in the good equilibrium is equal to α (recall that α is the fraction of banks hit by $\underline{\psi}$). If instead ψ_t^b could take more than two values, the measure of insolvent banks would be a more complicated function of the parameters.

Welfare. There is a welfare loss in the bad equilibria because of consumption misallocation across households. Optimality requires the same level of consumption for households with the same initial wealth. However, some households are last in line during a run, and thus their consumption expenditure is limited by the inability to withdraw money from their own banks. Other households are instead first in line during runs or face no runs on their own bank, so they can withdraw money at night and their consumption expenditure is higher.[28]

Solution method. I cannot solve for the bad equilibria in closed form, so I compute them numerically using the full nonlinear model. I conjecture that, at night, households run on banks hit by the shock $\underline{\psi}$. Under this conjecture, I solve for a candidate bad equilibrium using an approach

[26] Unspent money also includes the money withdrawn during runs by households with $\varepsilon_t^h = 0$.

[27] Note that both the price level p_t and the nominal price of capital Q_t drops in comparison to the good equilibrium. Solving the model for several values of the parameters, I find that the real price of capital in the bad equilibrium is greater than in the good equilibrium, $Q_t/p_t \geq Q^*/p^*$, even though Q_t/p_t and Q^*/p^* are very close to each other for reasonable parameter values.

[28] An additional welfare difference between good and bad equilibria is related to the distribution of wealth across households. This effect contributes to increasing welfare in the bad equilibria and arises because I impose a suboptimal banking contract. However, in a version of the model in which each household is part of a large family that pools the wealth of its members, as in Gertler and Kiyotaki (2015), this effect vanishes. In such an extension, consumption misallocation remains the only source of welfare difference, and thus welfare is always lower in the bad equilibrium.

Table 1: Parameter values

Parameter	Description	Value
β	Discount factor	0.988
Z	Productivity, $y(K) = ZK$	1/3
$\overline{M}$	Money supply	1
$\overline{K}$	Supply of capital	1
$\underline{\psi}$	Negative shock to capital	-0.25
$\overline{\psi}$	Positive shock to capital	0.03
α	Fraction of banks hit by the shock $\underline{\psi}$	0.1
κ	Pr $\left(\varepsilon_t^h = \bar{\varepsilon}\right)$	{0.5, 0.85}

based on the numerical computation of Gröbner bases. The candidate bad equilibrium is an equilibrium if $r_t\left(\underline{\psi}\right) < 0$, so that running is indeed optimal for households and the initial conjecture is verified. Appendix D provides the full list of the equations that characterize the equilibrium and more details about the computation.

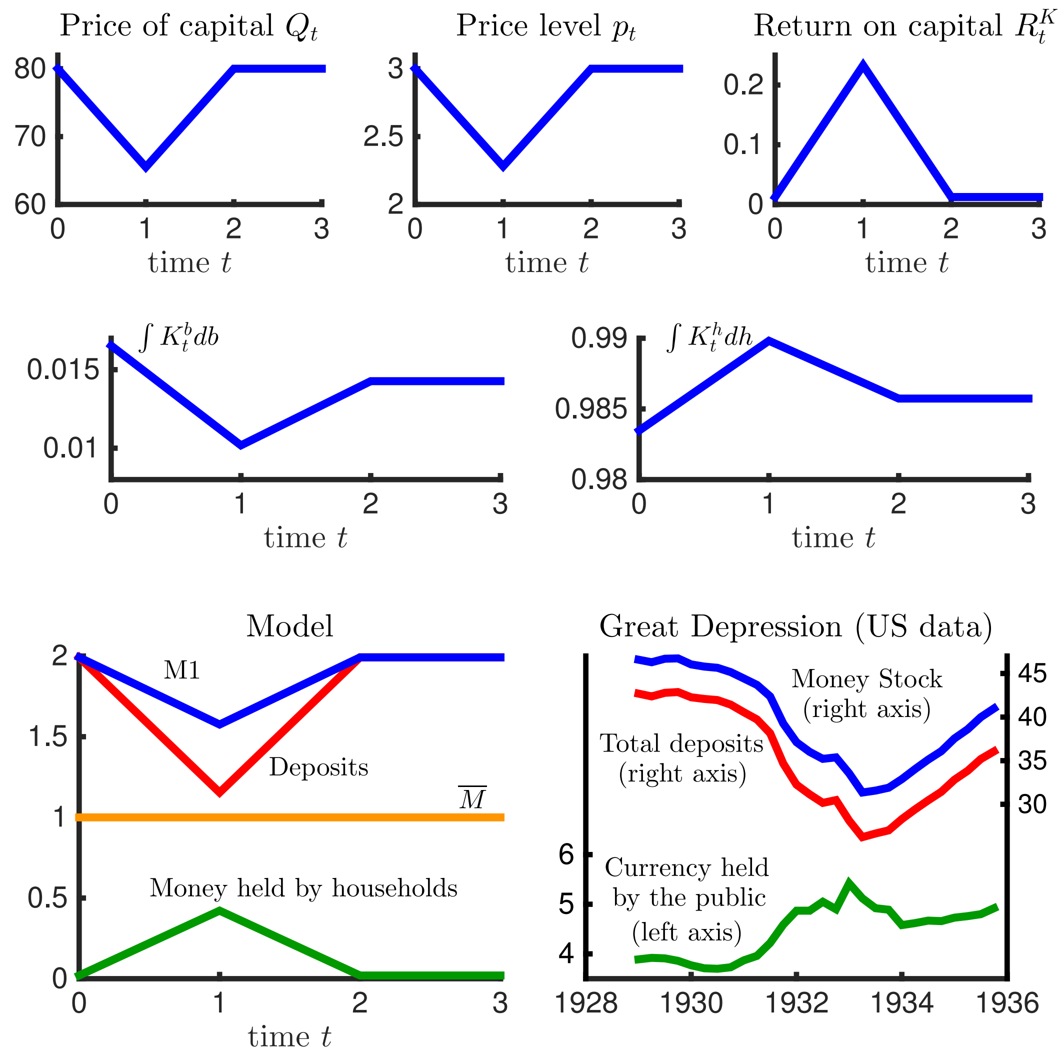
4.3 Numerical example

Figure 2 and Table 2 show the results of a numerically simulated example of the model. I focus on two parameterizations that differ in the value of κ (i.e., the probability that household h is hit by the preference shock $\bar{\varepsilon} > 0$), $\kappa = 0.5$ and $\kappa = 0.85$. I set $\alpha = 0.1$; therefore, 10% of the banks are hit by $\underline{\psi}$. The other parameters are presented in Table 1; the value of $\bar{\varepsilon}$ is determined by κ and the normalization in equation (2).

For $\kappa = 0.5$, the actual return on deposits of insolvent banks in the bad equilibrium is $r_t\left(\underline{\psi}\right) = -0.14 < 0$, and the other key endogenous variables are plotted in Figure 2. The economy is in steady state in $t = 0$, experiences a crisis in $t = 1$, and reverts to normal in $t = 2$. The top panel plots the prices Q_t, p_t, and the nominal return on capital R_t^K.

The middle panel plots the evolution of capital held by banks (left panel) and households (right panel). Two remarks related to the evolution of capital holdings are in order. First, during the crisis, banks have fewer resources because of the flight away from deposits. Therefore, banks reduce holdings of capital with respect to the pre-crisis level. Since the supply of capital is fixed, households must increase their holdings of capital in equilibrium. Second, after the crisis, at $t = 2$, banks' holdings of capital rebound, but they do not go back to the pre-crisis level; that is,

Figure 2: Bad equilibrium and comparison with the Great Depression



The economy experiences the crisis in period $t = 1$ and reverts to normal in $t = 2$. Top panel: prices (nominal price of capital Q_t, price level p_t, and return on capital R_t^K). Middle panel: stock of capital held by banks (left) and households (right) at the end of the day market. Bottom left panel: money market (money supply $\overline{M}$, deposits $\int_{\mathbb{H}} D_t^h dh$, money held by households $\int_{\mathbb{H}} M_t^h dh$, and $M1$ = deposits + money held by households). Parameter values: see Table 1, $\kappa = 0.5$.

Bottom right panel: based on Table 2 from Friedman and Schwartz (1970) ("Money Stock" is the sum of currency held by the public and deposits); data are quarterly and seasonally adjusted, in billions of dollars.

Table 2: Equilibria comparison, $\kappa = 0.85$

Variable		Good Equilibrium	Mild Crisis	Deep Crisis
Price level	p_t	3	2.79	2.59
Price of capital	Q_t	80	73	70.6
Money, households	$\int_{\mathbb{H}} M_t^h dh$	0	0.61	0.88
Deposits	$\int_{\mathbb{H}} D_t^h dh$	1.17	0.46	0.13
M1	$\int_{\mathbb{B}} D_t^b db + \int_{\mathbb{H}} M_t^h dh$	1.17	1.07	1.01
Return on capital	R_t^K	0.0125	0.11	0.14
Return deposits, b insolvent	r_t^b s.t. $N_t^b < 0$	(n.a.)	-0.13	-0.99

households hold more capital than at $t = 0$ and banks hold less capital. In other words, one effect of the crisis is to redistribute wealth in the economy from banks to households. This result is a consequence of the debt deflation; indeed, the denomination of deposits in nominal terms implies that the real value of households' preexisting deposits at the beginning of $t = 1$, d_1^h, increases.[29]

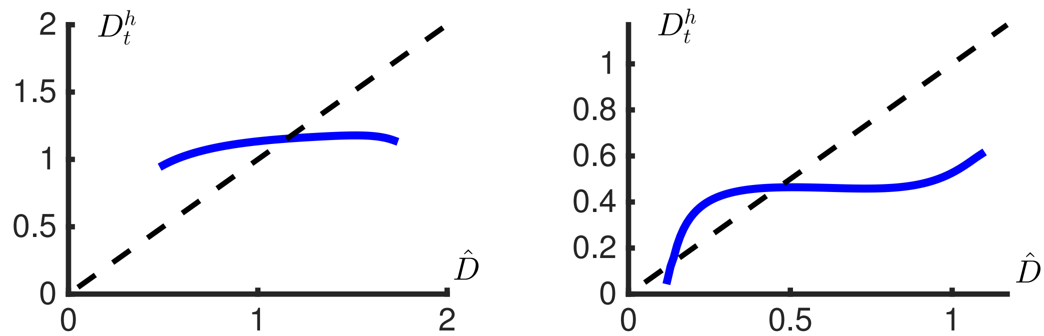
In addition, the numerical example demonstrates that sizable effects on prices and the money market can arise in equilibrium even if the size of the banking sector is very small. Figure 2 shows that banks hold between 1% and 2% of the total stock of capital;[30] yet, banking distress generates sizable general equilibrium effects in the bad equilibrium. This finding has implications for the analysis of the 2008 US financial crisis. Krishnamurthy et al. (2014) document that the contraction in an important part of the repo market backed by private asset-backed securities was small relative to the total stock of private asset-backed securities. Based on this fact, they argue that their evidence is against the view that a run on repo played a central role in the crisis. While my model does not directly address the run on repo, it nonetheless provides an example against the argument of Krishnamurthy et al. (2014) that a "small run" is inconsistent with a central role for such a run in a crisis.

The bottom left panel of Figure 2 plots some key variables in the money market: money held by households during the day, deposits, and a monetary aggregate denoted M1. M1 is defined as the sum of deposits and money held by households, in line with the standard definition of such a monetary aggregate. The model qualitatively replicates some key facts of the money market during

[29]He et al. (2010) find that securitized assets shifted from sectors dependent on repo financing to commercial banks. If the banking sector in the model is interpreted as the shadow banking system, and households in the model are interpreted as including both the US nonfinancial sector and commercial banks, the model is consistent with the evidence of He et al. (2010).

[30]Because of log utility, households want to set aside as deposits, in order to finance consumption, a fraction of wealth (approximately) equal to $1 - \beta$; since β is close to one, deposits are small compared to total wealth, and capital held by banks is just a small fraction of total capital in the economy. The same result is likely to hold in a richer calibrated version of the model.

Figure 3: Multiplicity of bad equilibria



Left panel: $\kappa = 0.5$; right panel: $\kappa = 0.85$. The dashed line is the 45 degree line.

the Great Depression, plotted in the bottom right panel of Figure 2. Both in the model and during the Great Depression, there is a flight to liquidity: a drop in the total stock of money (top line, M1 in the model), a drop in deposits (second line from the top), and an increase in currency held by the public (bottom line). Therefore, both in the model and during the Great Depression, there is a large drop in the money multiplier or, equivalently, a drop in velocity in the equation of exchange $\overline{M} \cdot velocity = p_t y\left(\overline{K}\right)$. A similar drop in money velocity took place during the 2008 crisis as well.[31]

Finally, Table 2 shows the result for $\kappa = 0.85$. Under this parameterization, there exist two bad equilibria that I label "mild crisis" and "deep crisis." The two bad equilibria are qualitatively identical, but in the deep crisis equilibrium, in comparison to the mild crisis, the drop in prices is more pronounced, the flight to liquidity and the drop in M1 are greater, and the return on deposits of insolvent banks is lower. The force that gives rise to multiple bad equilibria is analyzed in the next section.

4.4 Understanding the multiplicity of (bad) equilibria

The next proposition suggests that the driving force behind the multiplicity of bad equilibria and, more generally, behind the existence of (some) bad equilibria is a strategic complementarity across depositors. Recall that $\mathbb{H}(b)$ denotes the depositors of bank b, and thus $\int_{\mathbb{H}(b)} D_t^h dh = D_t^b$ is the amount of deposits of bank b.

Proposition 4.2. *Taking prices Q_t, p_t, and R_t^K as given, the actual return on deposits r_t^b of bank b with negative net worth ($N_t^b < 0$) satisfies $\partial r_t^b / \partial D_t^b > 0$.*

[31]In the United States, money velocity measured using the monetary base dropped dramatically, from about 17 in the second quarter of 2008 to about 8 in the first and second quarter of 2009. Calculations are based on the FRED database (St. Louis Fed Economic Data); velocity is computed as nominal GDP divided by the monetary base.

The proof is provided in Appendix B. To understand the result, recall that an insolvent bank b starts the period with negative net worth, $N_t^b < 0$. Such negative net worth implies that depositors of bank b must bear losses when the bank is liquidated, in $t+1$. The result $\partial r_t^b / \partial D_t^b > 0$ means that an increase in deposits at bank b implies that each dollar of deposit bears a small loss because the same negative net worth is spread across a large amount of deposits (i.e., the actual return on deposits r_t^b is higher). Consequently, the greater the deposits $\int_{\mathbb{H}(b)} D_t^h dh = D_t^b$ chosen by other depositors of bank b, the more willing household h is to hold deposits issued by bank b, explaining the strategic complementarity.

The result of Proposition 4.2 is derived by fixing prices and is thus a partial equilibrium exercise about one bank b in the economy. Next, I ask how each household's choice of deposits D_t^h responds to the choices of deposits made by all other households in the economy, taking into account general equilibrium effects. The answer is provided in Figure 3, which plots the best response of household h to the choices made by all other depositors in the economy.[32] That is, given a level of deposits $\hat{D}$ chosen by all other households in the economy (horizontal axis), Figure 3 plots the best response of household h.[33] Therefore, the intersection of the best response with the 45-degree line is a fixed point and thus an equilibrium.

Depending on the value of κ in Figure 3, there exist either one bad equilibrium or two bad equilibria. To understand this difference, recall that the maximization problem of banks, problem (5), includes the non-negativity constraint $K_t^b \geq 0$. In order for this constraint to be slack, deposits at banks must be sufficiently large so that banks have enough resources to hold $K_t^b \geq 0$ even if insolvent. For an insolvent bank with negative net worth $N_t\left(\underline{\psi}\right) < 0$, the constraint $K_t^b \geq 0$ is not binding if[34]

$$deposits \geq \frac{-N_t\left(\underline{\psi}\right)}{1-\kappa} > 0. \tag{24}$$

For the case $\kappa = 0.5$, equation (24) is satisfied if $deposits \geq 0.5$. Thus, it is not possible to fix a value of $\hat{D}$ lower than 0.5 when constructing Figure 3. For $\kappa = 0.85$, equation (24) is instead satisfied if $deposits \geq 0.135$ and the equilibrium value of deposits in the deep crisis equilibrium is 0.136.

Notably, the value of $N_t\left(\underline{\psi}\right)$ is endogenous and thus possibly affected by monetary policy. In the case with $\kappa = 0.5$, if a monetary injection improves the condition of banks by increasing the

[32] I use the term "best response" to emphasize that, even though the choices of household h depend on prices, prices are affected by the choices of all other households in the economy. Therefore, household h responds to the choices of other households, even if indirectly through prices.

[33] More precisely, Figure 3 is constructed by computing prices that would arise if the representative household in the economy is forced to choose deposits $\hat{D} \in (0, D^*)$ and by computing the optimal D_t^h chosen by one single household h facing such prices.

[34] Inequality (24) is obtained by combining the non-negativity constraint $K_t^b \geq 0$ with the budget constraint (6), the decision rule of banks $M_t^b = \kappa D_t^b$ (from Proposition 3.1), and the market-clearing condition for deposits in (20), and by focusing on a bank b with negative net worth, $N_t^b = N_t\left(\underline{\psi}\right) < 0$.

value of $N_t\left(\underline{\psi}\right)$, the term on the right-hand side of (24) decreases. As a consequence, the minimum threshold for deposits drops, and the possibility of second bad equilibrium opens up, even if such equilibrium does not exist without policy intervention.

5 Monetary policy

In this section, I study whether and how a central bank can eliminate the bad equilibria by injecting money into the economy. The central bank announces and fully commits to a policy that will be implemented during the day (when the Walrasian market is open) in the event of a panic. Under a credible commitment and an appropriate monetary policy, the bad equilibrium is eliminated, and thus the monetary injection is not implemented in equilibrium. I first extend the model to allow for monetary injections, and then I present the results.

5.1 Model with monetary injections

This section amends the baseline model to study the effect of a monetary injection. The central bank changes the money supply in the event of a sunspot shock at time t, setting $M_t = \overline{M}\left(1+\mu_t\right)$; the variable μ_t parameterizes the change in the money supply. I focus on temporary monetary injections, much like the policies implemented during the recent US financial crisis.[35] That is, the money supply reverts to $\overline{M}$ when the crisis is over at $t+1$.

Crucially, I examine two approaches that the central bank can use to inject money in the economy: *asset purchases* and *loans to banks*. In the first approach, the central bank buys capital on the Walrasian market at time t and then sells it at $t+1$. In the second approach, the central bank provides loans to private banks; the loans plus an interest rate are repaid at the beginning of $t+1$.

5.1.1 Central bank

In the event of a panic at time t, the central bank changes the money supply at time t to $M_t = \overline{M}\left(1+\mu_t\right)$. Since the stock of money at the beginning of time t is $\overline{M}$, the central bank injects new money $\mu_t\overline{M}$ during the day at time t. The central bank can inject the newly printed money using two tools: offering loans to banks in the amount L_t^{CB} and buying capital K_t^{CB} on the market (asset purchases) at the market price Q_t. Therefore,

$$L_t^{CB} + Q_t K_t^{CB} \leq \overline{M}\mu_t. \tag{25}$$

[35]In testimony before the Committee on Financial Services of the US House of Representatives, Bernanke (2010) suggests that the monetary expansions of the Federal Reserve are temporary: "In due course [...] as the expansion matures the Federal Reserve will need to begin to tighten monetary conditions."

Loans from the central bank to private banks must be equal to the demand from private banks in equilibrium:

$$L_t^{CB} = \int_{\mathbb{B}} L_t^b db, \tag{26}$$

where L_t^b are the loans demanded by each bank b. If a bank borrows L_t^b, it has to repay $L_t^b\left(1+R_t^{CB}\right)$ at $t+1$, where R_t^{CB} is an interest rate charged on loans. The central bank can choose either the quantity of loans L_t^{CB} or the price R_t^{CB}, but not both. I focus on the case in which the central bank chooses L_t^{CB} and the return R_t^{CB} is determined endogenously so that equation (26) holds, given banks' demand schedule for L_t^b.

If the central bank uses loans to banks, I assume that such loans are provided to all banks.[36] More importantly, the results crucially depend on whether the loans are senior in comparison to deposits or have the same seniority as deposits. If loans from the central bank are senior, they are repaid with certainty and the central bank does not face any loss. If the loans from the central bank have the same seniority as deposits, the central bank suffers losses on loans to banks that are insolvent.

At $t+1$, the money supply goes back to the pre-crisis level $\overline{M}$. The profits (or losses) of the central bank from this operation, denoted by T_{t+1}, are distributed to households. If loans to banks are senior to deposits:

$$T_{t+1} = L_t^{CB}\left(1+R_t^{CB}\right) + Q_t K_t^{CB}\left(1+R_t^K\right) - \mu_t \overline{M}. \tag{27}$$

The last term in equation (27) denotes the fact that the monetary injection is temporary; that is, the money $\overline{M}\mu_t$ injected at time t is taken back by the central bank at $t+1$. If instead loans to banks have the same seniority as deposits, transfers to households are

$$T_{t+1} = L_t^{CB}\left[\alpha\left(1+r_t\left(\underline{\psi}\right)\right) + (1-\alpha)\left(1+R_t^{CB}\right)\right] + Q_t K_t^{CB}\left(1+R_t^K\right) - \mu_t \overline{M}. \tag{28}$$

That is, a fraction α of loans are given to insolvent banks (i.e., to banks hit by the shock $\underline{\psi}$); therefore, the central bank gets the same return as depositors on these loans, given by $r_t\left(\underline{\psi}\right) < 0$. The remaining fraction $1-\alpha$ of loans is given to good solvent banks, and therefore the return on such loans is R_t^{CB}. The return on capital K_t^{CB} is the market return R_t^K, defined in equation (3).

5.1.2 Banks

In the extended model with monetary injections, the problem of banks is unchanged if the central bank uses only asset purchases. If instead the central bank injects some money using loans to

[36] That is, the central bank has the same information as the private sector and thus cannot distinguish between banks hit by $\overline{\psi}$ and banks hit by $\underline{\psi}$.

banks, each bank $b \in \mathbb{B}$ must choose its demand of loans from the central bank, denoted by L_t^b. As described in Section 5.1.1, loans L_t^b and the interest $1 + R_t^{CB}$ must be repaid to the central bank at $t + 1$. The budget constraint becomes

$$K_t^b Q_t + M_t^b \leq D_t^b + L_t^b + N_t^b. \tag{29}$$

The expressions that defines money and deposits at the end of the night at time t, m_t^b and d_t^b in (7), are replaced by

$$m_t^b \equiv \left(M_t^b - w_t^b\right) + y\left(K_t^b\right) p_t, \quad d_t^b \equiv \left(D_t^b - w_t^b\right)\left(1 + R_t^b\right) + L_t^b\left(1 + R_t^{CB}\right).$$

The results of Proposition 3.1 about D_t^b and M_t^b are unchanged; in particular, the bank still wants to hold an amount of money $M_t^b = \kappa D_t^b$ because none of the funds lent by the central bank will be withdrawn at night. The choice of capital is $K_t^b = \left(N_t^b + L_t^b + D_t^b - M_t^b\right)/Q_t$ and it follows from the budget constraint (29). Therefore, banks use all the resources borrowed from the central bank to buy capital because loans. The choice of L_t^b is similar to the choice of D_t^b and can be proven similarly:

$$L_t^b = \begin{cases} 0 & \text{if } R_t^{CB} > R_t^K \\ \text{any amount } \geq 0 & \text{if } R_t^{CB} = R_t^K \\ +\infty & \text{if } R_t^{CB} < R_t^K. \end{cases} \tag{30}$$

Since a generic bank b invests all the funds that it gets from the central bank in capital, the bank wants to hold $L_t^b = 0$ if the return paid to the central bank is greater than the return on capital ($R_t^{CB} > R_t^K$), $L_t^b = +\infty$ if the return paid to the central bank is less than the return on capital ($R_t^{CB} < R_t^K$), and any amount of loans if the two returns are equal ($R_t^{CB} = R_t^K$). In equilibrium, the return on the central bank's loans must equate the return on capital, $R_t^{CB} = R_t^K$, to make sure that banks are willing to take the amount of loans offered by the central bank.

Actual return on deposits and fraction of depositors served. If loans from the central bank are senior to deposits, the expression for $\widehat{r}_t^b$ in equation (10) is replaced by

$$1 + \widehat{r}_t^b = \frac{K_t^b\left(Q_{t+1} + Zp_t\right) - L_t^b\left(1 + R_t^{CB}\right)}{D_t^b - w_t^b}. \tag{31}$$

If instead loans to banks have the same seniority as deposits,

$$1 + \widehat{r}_t^b = \frac{K_t^b\left(Q_{t+1} + Zp_t\right)}{D_t^b - w_t^b + L_t^b}. \tag{32}$$

The fraction of depositors served during the run is unchanged and thus given by (11).

5.1.3 Households

Monetary injections do not affect the households' Bellman equation in (13). Nonetheless, the central bank might earn profits T_{t+1} on its operations, which are redistributed to households (see Section 5.1.1). Thus, in comparison to Section 3.2, the definition of wealth in $t+1$, equation (17), is replaced by

$$A_{t+1}^{h}\left(n_{t}^{h},\psi_{t+1}^{h}\right)=\left[K_{t}^{h}\left(1+\psi_{t+1}^{h}\right)\right]Q_{t+1}+d^{h}\left(n_{t}\right)+m^{h}\left(n_{t}^{h}\right)+\frac{A_{t}^{h}}{\overline{A}_{t}}T_{t+1} \tag{33}$$

where $\overline{A}_t=\int_{\mathbb{H}}A_t^h dh$ is the total wealth of the household sector. The formulation of equation (33) implies that transfers from the central bank are distributed proportionally to the wealth A_t^h of each household h. This assumption allows me to still be able to guess and verify the value function and to guess and verify that policy functions for choices during the day are proportional to wealth.

5.1.4 Market-clearing conditions and equilibrium

The market-clearing conditions for capital and money in (20) are replaced by:

$$\int_{\mathbb{B}}K_t^b db+\int_{\mathbb{H}}K_t^h dh+K_t^{CB}=\overline{K},\qquad \int_{\mathbb{B}}M_t^b db+\int_{\mathbb{H}}M_t^h dh=\overline{M}\left(1+\mu_t\right). \tag{34}$$

The other market-clearing conditions are unchanged.

The equilibrium concept is unchanged. However, the list of equilibrium objects also includes the return on loans from the central bank R_t^{CB}, choices of loans by each bank $\left\{L_t^b\right\}_{b\in\mathbb{B}}$, and the variables that describe monetary policy (money injected μ_t, loans L_t^{CB}, assets purchased K_t^{CB}, and transfers to depositors T_{t+1}). Moreover, in equilibrium, the budget constraint of the central bank (equation (25)) must hold, and the demand for loans $\int_{b\in\mathbb{B}}L_t^b db$ must be equal to the supply (equation (26)).

5.2 Monetary injections: eliminating the bad equilibria

The key result of the policy analysis is that loans to banks are a more effective tool than asset purchases for eliminating the crisis, provided that loans have the same seniority as deposits. That is, the size of the monetary injection required to eliminate the bad equilibrium is much smaller if the central bank uses loans to banks with the same seniority as deposits. If instead loans to banks have higher seniority than deposits, loans to banks are equivalent to asset purchases.

Table 3: Policy analysis

Central bank's tool	Monetary injection to eliminate bad equilibria	Q_t	$r_t(\underline{\psi})$	p_t
Asset purchases, $\kappa = .5$	+97.4%	$80 = Q^*$	-0.1	$2.97 < p^*$
Loans to banks, $\kappa = .5$ (same seniority as deposits)	+33.9%	$74 < Q^*$	0	$2.70 < p^*$
Asset purchases, $\kappa = .85$	+17.4%	$80 = Q^*$	-0.1	$2.99 < p^*$
Loans to banks, $\kappa = .85$ (same seniority as deposits)	+8%	$77 < Q^*$	0	$2.87 < p^*$

A sufficient condition for eliminating the bad equilibria is to achieve at least one of the following outcomes. First, if the central bank can sustain a price of capital $Q_t \geq Q^*$, Assumption 4.1 implies that all banks are solvent and thus the good equilibrium is the unique outcome. The central bank can indeed eliminate the bad equilibrium by sustaining $Q_t \geq Q^*$ because monetary injections create inflationary pressure, counteracting the drop in nominal prices – in particular, the nominal price of capital Q_t. Second, the central bank can eliminate the bad equilibrium by achieving a return on deposits $r_t^b \geq 0$ for all b. That is, if the return on deposits is positive for all banks, even those hit by $\underline{\psi}$, households prefer not to run at night; this is because the return from running and holding money is zero, whereas the return from not running and keeping the deposits in the banks is $r_t^b \geq 0$. With no runs at night, there is no incentive to fly to liquidity during the day, and thus the economy ends up in the good equilibrium.

Table 3 compares the ability of asset purchases and loans to banks with the same seniority as deposits to eliminate the bad equilibria. In the numerical example with $\kappa = 0.5$ (only one bad equilibrium under constant money supply), asset purchases are successful if the central bank almost doubles the money supply (+97.4%). With loans to banks, however, the money supply needs to increase only by 34%. In the numerical example with $\kappa = 0.85$ (two bad equilibria under constant money supply), a smaller increase in money is sufficient to eliminate a bad equilibrium. Nonetheless, the main result is unchanged; that is, the central bank can eliminate the bad equilibria by increasing the money supply by 17.4% with asset purchases or by 8% with loans to banks.

The results shown in the last three columns of Table 3 shed light on the channel that allows the central bank to eliminate the bad equilibria. These results are constructed as follows. I solve for the bad equilibria under a monetary injection that is smaller than but arbitrarily close to the level that eliminates the bad equilibria. As discussed above, monetary injections eliminate the bad equilibrium by achieving $Q_t \geq Q^*$ (and thus making banks solvent) or $r_t\left(\underline{\psi}\right) \geq 0$ (and thus eliminating the incentives of households to run at night). Under both calibrations ($\kappa = 0.5, 0.85$), asset purchases eliminate the bad equilibrium by sustaining the same price of capital that would prevail in the good equilibrium, $Q_t = Q^*$. The smallest monetary injection implemented with loans to banks with the same seniority as deposits, however, eliminates the incentives to run, $r_t\left(\underline{\psi}\right) = 0$. Moreover, the last column in Table 3 shows that the central bank can eliminate the bad equilibrium without the need to inflate the price level above the good equilibrium threshold p^*.

Note that, under asset purchases, the return on deposits $r_t\left(\underline{\psi}\right)$ remains negative, even if the central bank pushes Q_t arbitrarily close to Q^*. This is because asset purchases generate a general equilibrium feedback that reduces deposits D_t^h all the way to zero.[37] Thus, even if asset purchases increase $Q_t = Q^*$ and thus reduce the total losses of insolvent banks to zero, the reduction of D_t^h to zero implies that losses *per dollar of deposit* are not necessarily driven to zero.

What drives the difference between asset purchases and loans to banks that have the same seniority as deposits? If a monetary injection is implemented with loans to banks that have the same seniority as deposits, households are willing to hold more deposits than they are under asset purchases. This is because some of the losses of the insolvent banks will be borne by the central bank. This behavior of the household sector has two effects. First, from a partial equilibrium perspective, the actual return on deposits of insolvent banks $r_t\left(\underline{\psi}\right)$ is higher (see Proposition 4.2 for an explanation). Second, from a general equilibrium perspective, the strategic complementaries in the flight to liquidity are weakened or eliminated.

Finally, I analyze the case in which loans to banks are senior in comparison to deposits. In this situation, a monetary injection implemented with loans to banks is equivalent to an injection implemented with asset purchases. The result follows from the fact that the central bank does not bear any losses of insolvent banks in both cases. Different from the previous case, the central bank is able to recover the full value of loans, and thus depositors split the value of assets after the central bank is repaid. Proposition 5.1 formalizes this result. The proof is provided in Appendix B.

Proposition 5.1. *Given a policy $\mu_t > 0$ implemented with asset purchases, if there exists a bad equilibrium, then the same equilibrium exists in an economy in which the same policy is implemented using loans to banks with higher seniority than deposits.*

6 Conclusions

I have presented a new framework to analyze panic-driven bank runs in a dynamic general equilibrium model, and I have used it to study the ability of monetary injections to eliminate financial panics. Loans to banks with the same seniority as deposits are more effective at eliminating panics

[37] The reduction in deposits is a by-product of the higher price of capital Q_t, in comparison to a bad equilibrium with no policy intervention. Equation (3) implies that a higher Q_t reduces the return on capital, which is in turn used by banks to pay the return on deposits. Thus, the promised return on deposits R_t^D drops as well, and the demand for deposits is reduced. A related paper (Robatto, 2016) analyzes in more detail the effects of monetary injections on deposits during financial crises, using a simpler model of fundamentals-based runs.

than asset purchases are, in the sense that they require smaller monetary injections. This is because loans to banks with the same seniority as deposits imply that the central bank shares the losses of financial intermediaries with private households, thereby reducing the incentives to run.

This paper opens up several directions for future research. On the theoretical side, more work is required to identify the frictions that justify some of the assumptions that I have used, such as the nominal deposit contract. On the empirical side, the framework can be used for quantitative analysis of runs and financial crises. On the policy side, the framework model can be used to analyze other policies such as capital requirements.

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APPENDICES

Financial Crises and Systemic Bank Runs in a Dynamic Model of Banking

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March 9, 2017

A Extension: bankers and steady-state

The model described in the main text of the paper does not have a well-defined steady-state if banks have positive net worth, and it does not include a well-defined description of the ownership of banks. A minor extension to the model, presented in this Appendix, corrects for both shortcomings. This Appendix also characterizes the steady-state and the good equilibrium prices and quantities as a function of the parameters.

The lack of a well-defined steady-state in the main part of the paper is related to the profits made by banks. Due to perfect competition in the banking market, banks do not make any *extra profits*. Nonetheless, if $N_t^b > 0$, then bank b owns some of the capital stock that it manages; since capital pays a return, bank b gets the return on capital, and its net worth grows over time. If $R_t^D = R_t^K$ (which is the case in equilibrium), Proposition 3.1 implies that $\mathbb{E}_\psi\left(N_{t+1}^b\right) = N_t^b\left(1+R_t^K\right)$. To see this, rewrite N_{t+1}^b (equation (4) iterated one period forward) as:

$$\begin{aligned} N_{t+1}^b &= K_t^b\left(1+\psi_{t+1}^b\right)Q_{t+1} + m_t^b - d_t^b \\ &= K_t^b\left(1+\psi_{t+1}^b\right)Q_{t+1} + ZK_t^b p_t - (1-\kappa)\,D_t^b\left(1+R_t^D\right) \\ &= \left(1+\psi_{t+1}^b\right)\frac{N_t^b + (1-\kappa)\,D_t^b}{Q_t}\left(Q_{t+1}+Zp_t\right) - (1-\kappa)\,D_t^b\left(1+R_t^D\right) \end{aligned}$$

where the second line uses equation (7), and the third line uses Item 3 in Proposition 3.1. Taking expectations with respect to ψ_{t+1}^b:

$$\mathbb{E}_\psi\left(N_{t+1}^b\right) = \left[N_t^b + (1-\kappa)\,D_t^b\right]\underbrace{\left(\frac{Q_{t+1}+Zp_t}{Q_t}\right)}_{=1+R_t^K=1+R_t^D} - (1-\kappa)\,D_t^b\left(1+R_t^D\right)$$

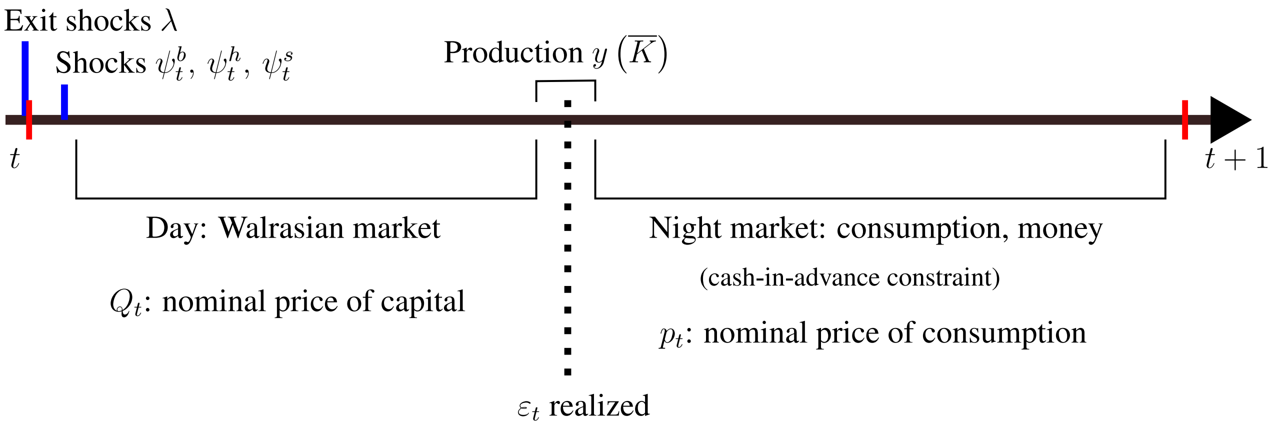
and the result follows by rearranging. Thus, abstracting from runs and bad equilibria, the net worth of banks increases over time.

Next, I introduce exit shocks that hit banks with probability λ, so that some banks are subject to liquidation every periods, paying some dividends. It is then possible to impose restrictions on λ so that the overall net worth of the banking sector remains constant over time. The model of Section 2 is a special case of the extension below, obtained by setting $\lambda = 0$.

A.1 Environment

The economy is populated by households (double continuum, indexed by $h \in \mathbb{H} = [0,1] \times [0,1]$); banks; and a unit mass of bankers. During the day, bankers get dividends π_t from banks (described below). Bankers are hand-to-mouth and use dividends π_t to finance consumption $\frac{\pi_t}{p_t}$ at

Figure 4: Timing of the model with exit shock



night.[38] Households preferences are the same as in Section 2.1. Markets and interaction between households and banks works as described in Section 2.1.

Banks and exit shocks. Between the night of $t-1$ and the day of t, each bank is subject to an i.i.d. exit shock with probability λ as represented in Figure 4. Assuming a law of large numbers, λ is also the fraction of banks hit by the exit shock. Exiting banks are liquidated while the day market is open at time t, together with banks that were subject to runs in $t-1$. Let $s \in \mathbb{S}$ denote banks that are liquidated in t (due to runs or exit shocks). Each surviving bank is instead "split" into $1/(1-\lambda)$ new banks in order to keep constant the measure of surviving banks. Let $b \in \mathbb{B} \equiv [0,1]$ denote active banks after this process.[39]

After exit shocks are realized and surviving banks are split, shocks to capital ψ_t^b and ψ_t^s are realized.

State variables of banks: law of motion. Consider bank b with capital, money, and deposits K_t^b, m_t^b, and d_t^b at the end of the night. If bank b is subject to a run in the night of time t, or subject to an exit shock at the end of time t, it becomes an exiting bank, denoted by $s \in \mathbb{S}$, with state:

$$\boldsymbol{X}_{t+1}^s = \left\{\left(K_t^b, m_t^b, d_t^b\right), \psi_{t+1}^s\right\} \tag{35}$$

[38] The assumption of hand-to-mouth bankers simplifies the analysis and the exposition, but the results are unchanged in a model without bankers, where banks pay dividends to households and households trade claims on the stream of dividends of banks.

[39] If some bank is subject to runs in $t-1$, so that the measure of $\mathbb{S}$ is larger than λ and the measure of banks after splitting is less than one, then new banks enter the market with no assets and no liabilities in order to keep the measure of active banks constant at one.

Otherwise, the bank is split into $1/(1-\lambda)$ new banks. The state variable of a bank b' that originates from the splitting of a bank b is:

$$\boldsymbol{X}_{t+1}^{b'} = \left\{ (1-\lambda) \left(K_t^b, m_t^b, d_t^b \right), \, \psi_{t+1}^{b'} \right\}. \tag{36}$$

Liquidation and dividends. Surviving banks operate as described in Section 2 of the text. Banks hit by the exit shock are liquidated when the day market is open at time t. Liquidation works as follows. All assets of the bank are sold on the market, and deposits that were not withdrawn at night are repaid (if the value of assets is insufficient, depositors are repaid pro-rata). Any value left after repaying depositors contributes to the total dividends π_t that are paid to bankers.

Given the vector of state variables $\{\boldsymbol{X}_t^s\}_{s\in\mathbb{S}}$ of banks under liquidation, the total value of dividends paid to bankers is:

$$\pi_t = \int_{\mathbb{S}} \max\{0, N_t^s\}\, ds. \tag{37}$$

where net worth N_t^s is computed according to equation (4).[40]

State of the economy. The aggregate state of the economy $\boldsymbol{X}_t$ at the beginning of the day is similar to the one defined in Section 2.4, but it also includes the state variables of banks under liquidation:

$$\boldsymbol{X}_t = \left\{ \Pr_t^B, \, s_t, \, \{\boldsymbol{X}_t^s\}_{s\in\mathbb{S}} \right\}.$$

Restriction on parameters. I impose restrictions on λ, $\underline{\psi}$, β, and κ to ensure that there exists a well-defined steady-state.

Assumption A.1. $\lambda = \frac{1-\beta}{\beta + (1-\beta)(1/\kappa)}$.

Assumption A.2. *The parameters* $\underline{\psi}$, β, *and* κ *satisfy* $1 + \beta\kappa\underline{\psi} + (1-\kappa)\left[\underline{\psi}(1-\beta) - \beta\right] > 0$.

A.2 Market clearing and equilibrium

Two of the market clearing conditions listed in Section 3.3 must be modified to account for dividends. In particular, the market clearing condition for money and consumption become, respectively:

$$\int_{\mathbb{B}} M_t^b db + \int_{\mathbb{H}} M_t^h dh = \overline{M} - \pi_t \tag{38}$$

[40] In the model of Section 2, which can be obtained setting $\lambda = 0$, only banks subject to runs are liquidated. In equilibrium, banks subject to runs (if any) are insolvent, and thus no dividend is paid in that case.

(because an amount π_t of money is used to pay dividends to bankers), and:

$$\int_{\mathbb{H}} c_t^h dh + \frac{\pi_t}{p_t} = Z\overline{K} \tag{39}$$

(because bankers are hand-to-mouth and their consumption is π_t/p_t).

The equilibrium definition is the same as Definition 3.5, with two modifications. First, the list of equilibrium objects includes also dividends π_t. Second, equilibrium dividends must satisfy equation (37).

The analysis of banks is the same as in Section 3.1,[41] and the analysis of households is the same as in Section 3.2.

A.3 Steady-state

First, I provide a definition of steady-state equilibrium (Definition A.3), and then I provide conditions on the state of the economy such that a steady-state equilibrium exists (Proposition A.4). Second, I characterize the unique steady-state equilibrium that satisfies Assumption 4.1 holding with equality (Proposition A.5). Third, I describe the state of the economy when banks are hit by the idiosyncratic shocks $\underline{\psi}$ and $\overline{\psi}$, and I show that there exists a good equilibrium, which is also a steady-state (Proposition A.6). Finally, I describe conditions on the state of the economy such that prices and quantities are the same as in a steady-state, but the economy is in steady-state from $t+1$ onward (Proposition A.7), which is relevant for the main analysis because this case encompasses what happens in $t+1$ if a bad equilibrium is realized at time t. Proofs are provided at the end.

Definition A.3. *(Steady-state equilibrium) Given $\boldsymbol{X}_t$, a steady-state equilibrium is an equilibrium such that:*

- *prices are:*

$$Q_t = Q^* \equiv \left[\frac{\beta}{1-\beta} + \left(\frac{1}{\kappa} - 1\right)\right] \frac{\overline{M}}{\overline{K}},$$

$$p_t = p^* = \frac{\overline{M}}{Z\overline{K}},$$

$$R_t^K = R_t^D = R^* \equiv \frac{(1-\beta)\,\kappa}{(1-\beta)\,(1-\kappa) + \beta\kappa}$$

[41] To be precise, the problem of bank should be written, instead of (5):

$$\max_{D_t^b, M_t^b, K_t^b} \mathbb{E}_{I,\,\psi}\left[\left(1 - I_{t+1}^b\right)\left(K_t^b\left(1 + \psi_t^b\right) Q_{t+1} + m_t^b - d_t^b\right) + I_{t+1}^b \max\left\{0,\, N_{t+1}^b\right\}\right]$$

because, if a bank is not hit by the exit shock λ, it maximizes the net value of assets (assets minus liabilities) before the splitting process. However, the results are unchanged.

- *dividends paid by exiting banks are* $\pi_t = \pi^*(\boldsymbol{X}_t) = \int_{\mathbb{S}} \left(K^s_{t-1}Q^* + m^s_{t-1} - d^s_{t-1}\right) ds$*;*
- *bank* $b \in [0,1]$ *has net worth* $N^b_t = K^b_{t-1}Q^* + m^b_{t-1} - d^b_{t-1} \geq 0$ *and choose* $D^b_t = D^*(\boldsymbol{X}_t) \equiv \frac{\overline{M} - \pi^*(\boldsymbol{X}_t)}{\kappa}$, $M^b_t = \kappa D^b_t = \overline{M} - \pi^*(\boldsymbol{X}_t)$ *and* $K^b_t = \left(N^b_t + \left(\overline{M} - \pi^*(\boldsymbol{X}_t)\right)\left(\frac{1-\kappa}{\kappa}\right)\right)(1/Q^*)$ *for all* b*;*
- *household* $h \in \mathbb{H}$ *has beliefs* $Pr^h_t\left(r^{b(h)}_t = R^*,\, l^h_t = +\infty\right) = 1$ *and its choice is given by Proposition 3.3 and* $\eta^M_t = 0$, $\eta^D_t = (1-\beta)/(1-\beta(1-\kappa))$, $\eta^K_t = \beta\kappa/(1-\beta(1-\kappa))$*;*
- *actual return on deposits and limits on withdrawals are* $r^b_t = R^*$ *for all* b *and* $l^h_t = +\infty$ *for all* h*;*
- *the state of the economy in* $t+1$ *is* $\boldsymbol{X}_{t+1} = \boldsymbol{X}_t$*.*

Proposition A.4 provides conditions for the existence of a steady-state equilibrium.

Proposition A.4. *(Existence of steady-state equilibrium) If Assumption A.1 holds and the state of the economy* $\boldsymbol{X}_t$ *satisfies:*

1. $Pr^B_t\left(\left\{\boldsymbol{X}^b_t \middle| K^b_{t-1}\left[\frac{\beta}{1-\beta} + \left(\frac{1}{\kappa} - 1\right)\right]\frac{\overline{M}}{\overline{K}} + m^b_{t-1} - d^b_{t-1} \geq 0\right\}\right) = 1$*;*
2. $m^b_{t-1} = K^b_t\left(\overline{M}/\overline{K}\right)$ *for all* $b \in \mathbb{B}$*;*
3. $d^b_{t-1} = \overline{M}(1-\kappa)\left[\overline{K}(1-\beta-\kappa+2\beta\kappa) - (1-\beta(1-\kappa))\int_{\mathbb{B}} K^b_t db\right] / \left(\overline{K}\beta\kappa^2\right) > 0$ *for all* $b \in \mathbb{B}$*;*
4. *the distribution* Pr^B_t *over the states of active banks is the same as the distribution over* $\{(1-\lambda)\boldsymbol{X}^s_t\}_{s\in\mathbb{S}}$*, and the measure of banks in* $\mathbb{S}$ *is* λ *(i.e., the set* $\mathbb{S}$ *includes only banks hit by the exit shock);*

then there exists a steady-state equilibrium.

Item 1 guarantees that all banks are solvent at the steady-state price $Q^* = \left[\frac{\beta}{1-\beta} + \left(\frac{1}{\kappa} - 1\right)\right]\frac{\overline{M}}{\overline{K}}$. Items 2 and 3 guarantee that the states $\left\{\boldsymbol{X}^b_t\right\}$ of active banks are constant over time. Item 4 guarantees that the states of exiting banks $\{\boldsymbol{X}^s_t\}$ are constant over time as well.

Next, I define the state of banks $\boldsymbol{X}^*$, and I show that if all banks have state $\boldsymbol{X}^*$ then a steady-state exists and Assumption 4.1 is satisfied. Moreover, no other steady-state satisfies Assumption 4.1 holding with equality. Let $\boldsymbol{X}^* \equiv \{(K^*, m^*, d^*), 0\}$ where:

$$K^* \equiv \frac{\overline{K}(1-\beta)(1-\kappa)}{1-\beta\left[1-\kappa\left(1+\underline{\psi}\right)\right]} \tag{40}$$

$$m^* \equiv K^*\left(\overline{M}/\overline{K}\right) \tag{41}$$

$$d^* \equiv \frac{\overline{M}(1-\kappa)\left[1+\underline{\psi}(1-\kappa) - \beta\left(1-\kappa+\underline{\psi}-2\kappa\underline{\psi}\right)\right]}{\kappa\left[1-\beta\left(1-\kappa\left(1+\underline{\psi}\right)\right)\right]}. \tag{42}$$

Proposition A.5. *(Main steady-state) If Assumptions A.1 and A.2 hold, and if the state of the economy satisfies* $Pr^B\left(\boldsymbol{X}^b_t = \boldsymbol{X}^*\right) = 1$, $\boldsymbol{X}^s_t = \frac{\boldsymbol{X}^*}{1-\lambda}$ *for all* $s \in \mathbb{S}$*, and the measure of banks in* $\mathbb{S}$ *is* λ*, there exists a unique steady-state equilibrium that satisfies Assumption 4.1.*

The next Proposition guarantees the existence of a good equilibrium when the one-time unanticipated idiosyncratic shocks to capital $\psi_t^i \in \left\{\underline{\psi},\, \overline{\psi}\right\}$ are realized, and the economy is (in $t-1$) in the steady-state analyzed by Proposition A.5.

Proposition A.6. *(Good equilibrium) If Assumptions A.1 and A.2 hold, and if the state of the economy satisfies:*

$$Pr_t^B\left(\boldsymbol{X}_t^b = \left\{\left(K^*,\, m^*,\, d^*\right),\, \underline{\psi}\right\}\right) = \alpha,$$
$$Pr_t^B\left(\boldsymbol{X}_t^b = \left\{\left(K^*,\, m^*,\, d^*\right),\, \overline{\psi}\right\}\right) = 1-\alpha,$$

$$\boldsymbol{X}_t^s = \left\{\left(\frac{K^*}{1-\lambda},\, \frac{m^*}{1-\lambda},\, \frac{d^*}{1-\lambda}\right),\, \underline{\psi}\right\} \quad \textit{for the fraction } \alpha \textit{ of banks under liquidation},$$

$$\boldsymbol{X}_t^s = \left\{\left(\frac{K^*}{1-\lambda},\, \frac{m^*}{1-\lambda},\, \frac{d^*}{1-\lambda}\right),\, \overline{\psi}\right\} \quad \textit{for the fraction } 1-\alpha \textit{ of banks under liquidation},$$

and the measure of banks in $\mathbb{S}$ is λ, there exists a steady-state equilibrium.

The last result of this Section encompasses the post-crisis case. After a bad equilibrium is realized, Proposition A.7 guarantees that the economy reaches a new steady-state.

Proposition A.7. *(Equilibrium after a crisis) If Assumption A.1 holds and the state of the economy satisfies:*

1. $Pr_t^B\left(\left\{\left.\boldsymbol{X}_t^b\right|\, K_{t-1}^b\left[\frac{\beta}{1-\beta}+\left(\frac{1}{\kappa}-1\right)\right]\frac{\overline{M}}{\overline{K}} + m_{t-1}^b - d_{t-1}^b \geq 0\right\}\right) = 1$;
2. *for each* $b \in \mathbb{B}$, $d_{t-1}^b > 0$ *or* $\left(K_{t-1}^b, m_{t-1}^b, d_{t-1}^b\right) = (0,0,0)$;
3. $\int_{\mathbb{S}} \max\left\{0,\, \left(K_{t-1}^s Q^* + m_{t-1}^s - d_{t-1}^s\right)\right\} ds = \frac{\lambda}{1-\lambda}\int_{\mathbb{B}}\left(K_{t-1}^b Q^* + m_{t-1}^b - d_{t-1}^b\right) db$;

then there exists an equilibrium with the same prices and quantities as the steady-state equilibrium but dividends that are defined as $\pi_t = \int_{\mathbb{S}} \max\left\{0,\, K_{t-1}^s Q^* + m_{t-1}^s - d_{t-1}^s\right\} ds$, and the economy is in a steady-state equilibrium from $t+1$ onward.

The conditions of Proposition A.7 are satisfied in $t+1$ if a crisis occurred at t. Item 1 holds because only good banks survive to $t+1$, while bad (insolvent) banks are subject to runs and liquidated. Item 2 holds because banks that originate from the splitting process have positive deposits, and new banks that enter the market to replace banks subjected to runs have zero assets and liabilities. Item 3 holds because, among the banks in $\mathbb{S}$ subject to liquidation, only banks hit by the exit shock have positive net worth (banks liquidated because of runs are insolvent), and their distribution is the same as active banks because the exit shocks are i.i.d.

Proofs.

Proof of Proposition A.4. Taking as given the return on deposits $R_t^D = R_t^K = R^*$, the choices of banks follow from Proposition 3.1. Item 1 guarantees that the net worth of banks is positive, using the definition of Q^*. Plugging banks' choices K_t^b and D_t^b (from Proposition 3.1) into equations (9) and (10), then $r_t^b = R_t^D = R^*$ because $N_t^b \geq 0$.

Choices of households for money, deposits, and capital are proportional to wealth (Proposition 3.3) and denoted by $M_t^h = \eta_t^M A_t^h$, $D_t^h = \eta_t^D A_t^h$, and $K_t^h = \eta_t^K A_t^h$. Using equation (61) (from the proof of Proposition 3.3, see Appendix B) and households beliefs (that trivially satisfy rationality), the variables η_t^M, η_t^D, and η_t^K solve:

$$\max_{\eta_t^M,\eta_t^D,\eta_t^K} \left\{ \kappa\bar{\varepsilon}\log\left(\frac{\eta_t^M+\eta_t^D}{p_t}\right) + \kappa\frac{\beta}{1-\beta}\log\left(\eta_t^K\left(1+R_t^K\right)\right) + \left[(1-\kappa)\frac{\beta}{1-\beta}\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^M+\eta_t^D\left(1+R^*\right)\right)\right]\right\}$$

subject to $\eta_t^M + \eta_t^D + \eta_t^K = 1$ and $\eta_t^M, \eta_t^D, \eta_t^K \in [0,1]$. First, I show that $M_t^h = 0$ or $\eta_t^M = 0$. Using the rearranged budget constraint $\eta_t^K = 1 - \eta_t^M - \eta_t^D$ and imposing $R_t^D = R_t^K = R^*$, the first-order conditions with respect to η_t^M and η_t^D are:

$$\text{FOC } \eta_t^M: \quad \frac{1}{\eta_t^D+\eta_t^M} - \underbrace{\frac{\beta R^*(1-\kappa)}{(1-\beta)\left[(1+R^*)\left(1-\eta_t^M\right)+\eta_t^M\right]}}_{>0} - \frac{\beta\kappa}{\left(1-\eta_t^D-\eta_t^M\right)(1-\beta)} \tag{43}$$

$$\text{FOC } \eta_t^D: \quad \frac{1}{\eta_t^D+\eta_t^M} - \frac{\beta\kappa}{\left(1-\eta_t^D-\eta_t^M\right)(1-\beta)} \tag{44}$$

Since η_t^D solves (44) equalized to zero, the FOC with respect to η_t^M is < 0. Thus it must be the case that $\eta_t^M = 0$ or $M_t^h = 0$ due to the non-negativity constraint on money. Moreover, using $\eta_t^M = 0$, equation (44) can be solved for η_t^D:

$$\eta_t^D = \frac{1-\beta}{1-\beta(1-\kappa)} \tag{45}$$

and the value of η_t^K is computed using $\eta_t^M + \eta_t^D + \eta_t^K = 1$. Since all banks are solvent and pay the return $r_t^b = R_t^D = R^* > 0$ and $l_t^h = +\infty$, the optimal withdrawal decision at night is to withdraw only if $\varepsilon_t^h = \bar{\varepsilon}$ (Proposition 3.3). As only impatient households withdraw, and banks hold $M_t^b = \kappa D_t^b$, then there are no runs and $l_t^h = +\infty$.

The expression for dividends, equation (37), is trivially satisfied. It is useful, for future reference, to rewrite π_t as:

$$\begin{aligned}\pi_t = \pi^*(\boldsymbol{X}_t) &= \int_{\mathbb{S}}\left(K_{t-1}^s Q^* + m_{t-1}^s - d_{t-1}^s\right)ds \\ &= \frac{\lambda}{1-\lambda}\int_{\mathbb{B}}\left(K_{t-1}^b Q^* + m_{t-1}^b - d_{t-1}^b\right)db\end{aligned} \tag{46}$$

where the last line follows from Item 4 and from the fact that the measure over $\mathbb{B} = [0,1]$ is one, while the measure over $\mathbb{S}$ is λ.

Next, I check that market clearing conditions hold. Using the choices of banks in the definition of steady-state, money held by banks is:

$$\begin{aligned}\int_{\mathbb{B}} M_t^b db &= \int_{\mathbb{B}} \kappa D_t^b db \\ &= \overline{M} - \pi^* (\boldsymbol{X}_t) \end{aligned} \tag{47}$$

and $\int_{\mathbb{H}} M_t^h dh = 0$. Thus money market clearing (38) holds.

To check the market clearing condition for deposits, I need to compute D_t^h, which requires knowing the value of households' wealth. Using Corollary 3.4, I compute the wealth $\overline{A}_t$ of the representative household summing the value of capital, money, and deposits of households. The wealth $\overline{A}_t$ is:

$$\overline{A}_t = \left(\overline{K} - \frac{\int_{\mathbb{B}} K_{t-1}^b db}{1-\lambda} \right) Q^* + \left(\overline{M} - \frac{\int_{\mathbb{B}} m_{t-1}^b db}{1-\lambda} \right) + \frac{\int_{\mathbb{B}} d_{t-1}^b db}{1-\lambda}. \tag{48}$$

The first term on the right-hand side is the total supply of capital $\overline{K}$ minus the capital owned by active banks $\int_{\mathbb{B}} K_{t-1}^b db$ minus the capital owned by banks in liquidation $\int_{\mathbb{S}} K_t^s ds = \frac{\lambda}{1-\lambda} \int_{\mathbb{B}} K_{t-1}^b db$ (which follows from Item 4); thus $\overline{K} - \int_{\mathbb{B}} K_{t-1}^b db - \frac{\lambda}{1-\lambda} \int_{\mathbb{B}} K_{t-1}^b db = \frac{\int_{\mathbb{B}} K_{t-1}^b db}{1-\lambda}$. The second and third expressions on the right-hand side (money and deposits) are computed similarly to capital. Item 3 is sufficient to guarantee that households hold a strictly positive amount of deposits and capital (for capital, the result follows using the numerator in Item 3), and thus of money too (Item 2). To verify the market clearing condition for deposits in (20), I need to check that:

$$0 = \int_{\mathbb{H}} D_t^h dh - \int_{\mathbb{B}} D_t^b db = \eta_t^D \overline{A}_t - \frac{\overline{M} - \pi^* (\boldsymbol{X}_t)}{\kappa} \tag{49}$$

where the last equality uses $\int_{\mathbb{H}} D_t^h dh = \eta_t^D \int_{\mathbb{H}} A_t^h dh = \eta_t^D \overline{A}_t$ and the value of deposits of banks D_t^b in the definition of steady-state. Plugging (46) and (48) into (49) and rearranging, it is possible to verify that (49) holds.

In the goods market, total expenditure is the sum of money spent by households and bankers

$\int_{\mathbb{H}} p_t c_t^h dh + \pi_t$:

$$\begin{aligned}
\int_{\mathbb{H}} p_t c_t^h dh + \pi_t &= \int_{\mathbb{H}} w_t^h + \pi^* \left(\boldsymbol{X}_t\right) \\
&= \kappa \int_{\mathbb{H}} D_t^h + \pi^* \left(\boldsymbol{X}_t\right) \\
&= \kappa \int_{\mathbb{B}} D_t^b + \pi^* \left(\boldsymbol{X}_t\right) = \overline{M}
\end{aligned}$$

where the first and second equalities use Proposition 3.3 and $M_t^h = 0$ for all h; the third equality uses market clearing for deposits; and the last equality uses the previous result about the money market, equation (47). Therefore all the money supply $\overline{M}$ is spent, and thus the total demand for consumption goods is $\frac{\overline{M}}{p^*} = Z\overline{K}$, where the equality uses the definition of p^*. Since $Z\overline{K}$ is also the supply of consumption goods, market clearing holds in this market as well.

The market clearing for capital holds by Walras' Law.

Finally, I show that the economy is in steady-state. Deposits at the end of the night, before the exit shocks, are:

$$\begin{aligned}
d_t^b &= \left(D_t^b - w_t^b\right)\left(1 + R^*\right) \\
&= \left(D_t^b - \kappa D_t^b\right)\left(1 + R^*\right) \\
&= \frac{\overline{M} - \pi^* \left(\boldsymbol{X}_t\right)}{\kappa} \left(1 - \kappa\right)\left(1 + R^*\right) \\
&= \frac{\overline{M} - \frac{\lambda}{1-\lambda} \int_{\mathbb{B}} \left(K_{t-1}^b Q^* + m_{t-1}^b - d_{t-1}^b\right) db}{\kappa} \left(1 - \kappa\right)\left(1 + R^*\right) = \left(\frac{1}{1-\lambda}\right) d_{t-1}^b
\end{aligned}$$

where the second equality uses $w_t^b = \int_{\mathbb{H}(b)} w_t^h dh = \kappa D_t^b$; the third equality uses $D_t^b = \frac{\overline{M} - \pi^*(\boldsymbol{X}_t)}{\kappa}$ in the definition of steady-state; the fourth equality uses equation (46); and the last equality uses Assumption A.1 about λ, Items 2 and 3, and rearranges. Money at the end of the night, before exit shocks, is:

$$\begin{aligned}
m_t^b &= p^* Z K_t^b \\
&= \frac{\overline{M}}{Z\overline{K}} Z \left(\frac{K_{t-1}^b Q^* + m_{t-1}^b - d_{t-1}^b + (1-\kappa)\frac{\overline{M} - \pi^*(\boldsymbol{X}_t)}{\kappa}}{Q^*}\right) \\
&= \frac{\overline{M}}{\overline{K}} \left(\frac{K_{t-1}^b \left[\frac{\beta}{1-\beta} + \left(\frac{1}{\kappa} - 1\right)\right] \frac{\overline{M}}{\overline{K}} + K_{t-1}^b \frac{\overline{M}}{\overline{K}} - \frac{\overline{M} - \pi^*(\boldsymbol{X}_t)}{\kappa}(1-\kappa) + (1-\kappa)\frac{\overline{M} - \pi^*(\boldsymbol{X}_t)}{\kappa}}{\left[\frac{\beta}{1-\beta} + \left(\frac{1}{\kappa} - 1\right)\right] \frac{\overline{M}}{\overline{K}}}\right) \\
&= \left(\frac{1}{1-\lambda}\right) K_{t-1}^b \frac{\overline{M}}{\overline{K}}
\end{aligned}$$

where the second equality uses the definition of $p^*$and the expression for K_t^b in the definition of steady-state; the third equality uses the definition of Q^* and Items 2 and 3; and the last equality rearranges using Assumption A.1 about λ. Similarly, we can compute:

$$\begin{aligned} K_t^b &= \frac{K_{t-1}^b Q^* + m_{t-1}^b - d_{t-1}^b + (1-\kappa)\frac{\overline{M}-\pi^*(\boldsymbol{X}_t)}{\kappa}}{Q^*} \\ &= \left(\frac{1}{1-\lambda}\right) K_{t-1}^b. \end{aligned}$$

Then, using the law of motion of the state vector of bank b' that originates from the splitting of bank b, equation (36):

$$\begin{aligned} \boldsymbol{X}_{t+1}^{b'} &= \left\{(1-\lambda)\left(K_t^b, m_t^b, d_t^b\right),\, \psi_{t+1}^{b'}\right\} \\ &= \left\{\left(K_{t-1}^b, m_{t-1}^b, d_{t-1}^b\right),\, \psi_{t+1}^{b'}\right\} \end{aligned}$$

which is equal to $\boldsymbol{X}_t^b$ (since $\psi_{t+1}^{b'} = 0$ with probability one). Moreover, $\left\{\boldsymbol{X}_{t+1}^s\right\}_{s\in\mathbb{S}} = \left\{\boldsymbol{X}_t^s\right\}_{s\in\mathbb{S}}$ follows from Item 4, from the laws of motion of the states of banks, equations (35) and (36), and from the i.i.d. assumption of the exit shocks. □

Proof of Proposition A.5. The state of the economy satisfies all the conditions of Proposition A.4. In particular, Item 1 of Proposition A.4 follows from the definition of $\boldsymbol{X}^*$; Item 2 of Proposition A.4 holds trivially; Item 3 of Proposition A.4 follows from the definition of $\boldsymbol{X}^*$, and $d^* > 0$ due to Assumption A.2; Item 4 of Proposition A.4 holds trivially. Therefore, there exists a steady-state equilibrium. Moreover, $\boldsymbol{X}^*$ satisfies Assumption 4.1. Uniqueness can be shown as follow. Combining equation (23) holding with equality, the fact that all banks must be alike (Assumption 4.1), and Items 2 and 3 from Proposition A.4, finding the state of the economy that gives rise to a steady-state equilibrium reduces to solving one linear equation, (23) holding with equality, in one unknown (the capital of bank b). Therefore the solution is unique. □

Proof of Proposition A.6. The conditions of Proposition A.4 are satisfied and can be checked as described in the Proof of Proposition A.5. □

Proof of Proposition A.7. The proof is very similar to the proof of Proposition A.4, with few differences. First, equation (46) is derived using Item 3 of Proposition A.7. Second, Item 2 of Proposition A.7 guarantees that $\overline{A}_t > 0$. Third, $\boldsymbol{X}_{t+1}$ is not necessarily equal to $\boldsymbol{X}_t$. As all the choices of banks at time t are the same as in steady-state, then K_t^b, m_t^b, and d_t^b are the same as in a steady-state equilibrium. Therefore $\left\{\boldsymbol{X}_{t+1}^b\right\}_{b\in\mathbb{B}}$ satisfies Items 1, 2, and 3 of Proposition A.4. $\boldsymbol{X}_{t+1}$ satisfies also Item 4 of Proposition A.4 due to the i.i.d. assumption of the exit shocks. □

B Proofs

Proof of Proposition 3.1. Since the shock $\psi^b_{t+1} = 0$ with probability one, I will ignore ψ^b_{t+1} and, where relevant, equalities should be interpreted in the almost sure sense.

I first focus on the case $N^b_t \geq 0$ and I guess that $N_{t+1} \geq 0$; thus $\max\{0,\ N_{t+1}\} = N_{t+1}$, therefore problem (5) collapses to (8). Thus, the bank will be solvent in $t+1$, and it will have enough resources to repay depositors because equations (9) and (10) imply that $r^b_t = R^D_t$. As a result, only impatient households (households with preference shock $\varepsilon^h_t = \bar{\varepsilon}$) will withdraw at night, and by the law of large numbers assumed in Section 2.1, a fraction κ of depositors are impatient. Therefore, bank b chooses $M^b_t = \kappa D^b_t$ in order to have enough money to repay depositors. Withdrawals are given by $w^b_t = \kappa D^b_t$, and they satisfy the feasibility constraint $w^b_t \leq M^b_t$.

Using the definition of net worth N^b_{t+1}, $M^b_t = w^b_t = \kappa D^b_t$, the definition of R^K_t in equation (3), the budget constraint (6), and the definitions of m^b_t and d^b_t in (7), the problem of banks becomes:

$$\max_{D^b_t} \left[D^b_t (1-\kappa)\left(R^K_t - R^D_t\right) + N^b_t\left(1 + R^K_t\right)\right].$$

The objective function is linear in D^b_t, so the solution depends on the sign of $R^K_t - R^D_t$. If $R^D_t > R^K_t$, then $D^b_t = 0$. If $R^D_t < R^K_t$, then $D^b_t = +\infty$. If $R^D_t = R^K_t$, any amount $D^b_t \geq 0$ is a solution. To verify the guess $N^b_{t+1} \geq 0$, note that if $R^K_t < R^D_t$ then $D^b_t = 0$, $M^b_t = \kappa D^b_t = 0$, and $N^b_{t+1} = N^b_t\left(1 + R^K_t\right) > 0$. If instead $R^K_t > R^D_t$, the bank issues $D^b_t = +\infty$; therefore, $N^b_{t+1} = +\infty$. Finally, if $R^K_t = R^D_t$, then $N^b_{t+1} = N^b_t\left(1 + R^K_t\right) > 0$ as well. Thus, the guess is verified.

If $N^b_t < 0$ and $R^D_t < R^K_t$, the bank can achieve $N^b_{t+1} = +\infty$ by issuing $D^b_t = +\infty$. Therefore, guessing $N^b_{t+1} > 0$ and following the same steps as before allows us to verify the guess.

To complete the proof, I have to analyze the case $N^b_t < 0$ and $R^D_t \geq R^K_t$. To discuss this case, let me first state and prove an intermediate result.

Lemma B.1. *If $N^b_t < 0$ and $R^D_t \geq R^K_t \geq 0$, then $N^b_{t+1} < 0$ and $r^b_t < R^D_t$ for any feasible choice $\left\{D^b_t, M^b_t, K^b_t\right\}$.*

Proof. Using equation (4) iterated one period forward, the net worth at $t+1$ is:

$$\begin{aligned} N^b_{t+1} &= Q_t K^b_t \left(\frac{Q_{t+1} + Z p_t}{Q_t}\right) + \left(M^b_t - w^b_t\right) - \left(D^b_t - w^b_t\right)\left(1 + R^D_t\right) \\ &= N^b_t\left(1 + R^K_t\right) + \left(D^b_t - M^b_t\right)\left(1 + R^K_t\right) + \left(M^b_t - w^b_t\right) - \left(D^b_t - w^b_t\right)\left(1 + R^D_t\right) \quad (50) \\ &= \underbrace{N^b_t\left(1 + R^K_t\right)}_{<0} + R^K_t\left(D^b_t - M^b_t\right) - R^D_t\left(D^b_t - w^b_t\right) < 0 \end{aligned}$$

where the first equality uses equation (7), the production technology $y\left(K_t^b\right) = ZK_t^b$, and rearranges; the second equality uses the definition of R_t^K in equation (3), the budget constraint (6), and the assumption of the Proposition that that the non-negativity constraint on capital does not bind; the third equality rearranges. Since feasibility requires $w_t^b \leq M_t^b$, then $D_t^b - M_t^b \leq D_t^b - w_t^b$. Note that $D_t^b - M_t^b > 0$ using the budget constraint (6) and the assumption $N_t^b < 0$. Therefore, using the assumption $0 \leq R_t^K \leq R_t^D$ of the Lemma, we can write $R_t^K\left(D_t^b - M_t^b\right) - R_t^D\left(D_t^b - w_t^b\right) \leq 0$ and the inequality in the last line of (50) follows. Then, using the first line of (50):

$$K_t^b\left(Q_{t+1} + Zp_t\right) + \left(M_t^b - w_t^b\right) - \left(D_t^b - w_t^b\right)\left(1 + R_t^D\right) < 0 \tag{51}$$

and using $w_t^b \leq M_t^b$ and comparing (51) with (9) and (10), we conclude $r_t^b < R_t^D$. □

As a consequence of Lemma B.1, a bank b with $N_t^b < 0$ will have $N_{t+1}^b < 0$ and will be liquidated in $t+1$ (due to the assumption that $r_t^b < R_t^D$ implies that bank b will be liquidated in $t+1$). Therefore, $I_{t+1}^b = 1$ in problem (5), and problem (5) collapses to (8) even in this case. An insolvent bank (net worth $N_t^b < 0$) is indifferent among any choice because $\max\left\{0, N_{t+1}^b\right\} = 0$ for any feasible choice $\left\{D_t^b, M_t^b, K_t^b\right\}$. Thus, it is (weakly) optimal to take the choices described by Proposition 3.1, in the sense that no deviation from those choices is profitable. □

Proof of Proposition 3.3. I guess that:

- the value function has the form $V_t\left(A_t^h\right) = \frac{1}{1-\beta}\log A_t^h + \Xi_t$ where Ξ_t is independent of A_t^h;
- the policy functions are $M_t^h = \eta_t^M A_t^h$, $D_t^h = \eta_t^D A_t^h$ and $K_t^h = \frac{\eta_t^K A_t^h}{Q_t}$ where η_t^M, η_t^D, and $\eta_t^K \in [0,1]$ are independent of A_t^h and $\eta_t^M + \eta_t^D + \eta_t^K = 1$.

To verify the guess and prove the other results of the Proposition, I proceed backward. I analyze first the problem at night, i.e., choices of withdrawals $w^h\left(n_t^h\right)$ and consumption $c^h\left(n_t^h\right)$.

If $\varepsilon_t^h = \bar{\varepsilon}$, I can rewrite the right-hand side of the Bellman equation (13) and omit terms that do not depend on choices taken at night when $\varepsilon_t^h = \bar{\varepsilon}$. I use the notation $n_t^h|\bar{\varepsilon}$ to denote a state of the world $n_t^h \in \mathcal{N}$ for household h, conditional on $\varepsilon_t^h = \bar{\varepsilon}$. Using the assumption that the shock $\psi_{t+1}^h = 0$ with probability one, equation (17) and the guess about the value function:

$$\max_{w^h\left(n_t^h|\bar{\varepsilon}\right), c^h\left(n_t^h|\bar{\varepsilon}\right)} \left[\bar{\varepsilon}\log c^h\left(n_t^h|\bar{\varepsilon}\right) + \frac{\beta}{1-\beta}\log\left(K_t^h Q_{t+1} + d^h\left(n_t^h|\bar{\varepsilon}\right) + m^h\left(n_t^h|\bar{\varepsilon}\right)\right)\right] \tag{52}$$

subject to the cash-in-advance constraint and the constraint on withdrawals:

$$p_t c^h\left(n_t^h|\bar{\varepsilon}\right) \leq M_t^h + w^h\left(n_t^h|\bar{\varepsilon}\right) \tag{53}$$

$$0 \leq w^h\left(n_t^h|\bar{\varepsilon}\right) \leq \min\left\{D_t^h, l_t^h\right\} \tag{54}$$

where, similarly to (18) and (19):

$$d^h\left(n_t^h|\bar{\varepsilon}\right) \equiv \left[D_t^h - w^h\left(n_t^h|\bar{\varepsilon}\right)\right]\left(1 + r_t^{b(h)}\right) \tag{55}$$

$$m^h\left(n_t^h|\bar{\varepsilon}\right) \equiv \left[M_t^h + w^h\left(n_t^h|\bar{\varepsilon}\right) - p_t c^h\left(n_t^h|\bar{\varepsilon}\right)\right] + p_t ZK_t^h. \tag{56}$$

I argue that the cash-in-advance constraint (53) must be satisfied with equality; otherwise the household would be better off by changing its decisions during the day. If there is unspent cash when $\varepsilon_t^h = \bar{\varepsilon}$ the household could reduce its holdings of money M_t^h and its withdrawals (thus, reduce deposits D_t^h) and invest more in capital. Therefore equations (53) and (56) become:

$$c^h\left(n_t^h|\bar{\varepsilon}\right) = \frac{M_t^h + w^h\left(n_t^h|\bar{\varepsilon}\right)}{p_t} \tag{57}$$

and

$$m^h\left(n_t^h|\bar{\varepsilon}\right) = p_t\left(ZK_t^h\right) \tag{58}$$

Plugging (55), (57), and (58) into (52), using the guesses about M_t^h, D_t^h, and K_t^h, and using the definition of R_t^K in equation (3):

$$\max_{w^h\left(n_t^h|\bar{\varepsilon}\right)}\left[\bar{\varepsilon}\log\left(\frac{\eta_t^M A_t^h + w^h\left(n_t^h|\bar{\varepsilon}\right)}{p_t}\right)\right.$$
$$\left. + \frac{\beta}{1-\beta}\log\left(\eta_t^K A_t^h\left(1+R_t^K\right) + \left[\eta_t^D A_t^h - w^h\left(n_t^h|\bar{\varepsilon}\right)\right]\left(1 + r_t^{b(h)}\right)\right)\right] \tag{59}$$

subject to the constraint on withdrawals (54).

I claim that, if $l_t^h = +\infty$, then $w^h\left(n_t^h|\bar{\varepsilon},\, l_t^h = +\infty\right) = D_t^h = \eta_t^D A_t^h$. Since the actual return on deposits $r_t^{b(h)}$ is weakly lower than the promised return R_t^D, i.e., $r_t^{b(h)} \leq R_t^D$, and since the Proposition considers the case $R_t^D = R_t^K$, then $r_t^{b(h)} \leq R_t^K$. That is, the return on deposits is always weakly lower than the return on capital. Using also Assumption 3.2, households decide to buy deposits only to finance consumption expenditures in case their realized preference shock is $\varepsilon_t^h = \bar{\varepsilon}$. Therefore, for the case $\varepsilon_t^h = \bar{\varepsilon}$ and $l_t^h = +\infty$, the entire amount of deposits D_t^h is withdrawn and used for deposits.

If $l_t^h = 0$, the bank has no cash when household h is served, so withdrawals are constrained to be zero, $w^h\left(n_t^h|\bar{\varepsilon},\, l_t^h = 0\right) = 0$.

If the preference shock is zero, $\varepsilon_t^h = \underline{\varepsilon} = 0$, then $c^h\left(n_t^h|\underline{\varepsilon}\right) = 0$ since the household gets no utility from consumption. The right-hand side of the Bellman equation (13) can be written, similarly to equation (59):

$$\max_{w^h\left(n_t^h|\underline{\varepsilon}\right)} \log\left(\eta_t^K A_t^h\left(1+R_t^K\right)+\eta_t^M A_t^h+w^h\left(n_t^h|\underline{\varepsilon}\right)+\left[\eta_t^D A_t^h-w^h\left(n_t^h|\underline{\varepsilon}\right)\right]\left(1+r_t^{b(h)}\right)\right)$$

subject to $0 \leq w^h\left(n_t^h|\underline{\varepsilon}\right) \leq \min\left\{D_t^h,\, l_t^h\right\}$. Taking a monotonic transformation of the objective function (to get rid of the log) and omitting terms that do not depend on $w^h\left(n_t^h|\underline{\varepsilon}\right)$:

$$\max_{w^h\left(n_t^h|\underline{\varepsilon}\right)} -w^h\left(n_t^h|\underline{\varepsilon}\right) r_t^{b(h)} \quad \text{s.t.} \quad 0 \leq w^h\left(n_t^h|\underline{\varepsilon}\right) \leq \min\left\{D_t^h,\, l_t^h\right\}.$$

Because of the linearity, the solution is always at the corner, i.e., either zero or $\min\left\{D_t^h,\, l_t^h\right\}$:

$$w^h\left(n_t^h|\underline{\varepsilon}\right)=\begin{cases} D_t^h=\eta_t^D A_t^h & \text{if } r_t^{b(h)}<0 \text{ and } l_t^h=+\infty \\ 0 & \text{if } r_t^{b(h)}<0 \text{ and } l_t^h=0 \\ 0 & \text{if } r_t^{b(h)}\geq 0 \text{ and } l_t^h\in\{0,+\infty\}. \end{cases}$$

Note that if $r_t^{b(h)} < 0$, the return on deposits not withdrawn is negative and thus lower than the return on money (the return on money is zero). If $r_t^{b(h)} < 0$, household h runs to withdraw as much money as possible, provided that the household is able to reach its own bank while the bank still has money to pay withdrawals.

To sum up, choices at night can be classified into four cases.

1. $\varepsilon_t^h = \bar{\varepsilon}$, $r_t^{b(h)} \in \mathbb{R}$ and $l_t^h = +\infty$. Withdrawals are $w_t^h = D_t^h$, and consumption is $c_t^h = \frac{M_t^h + D_t^h}{p_t}$;
2. $\varepsilon_t^h = \bar{\varepsilon}$, $r_t^{b(h)} \in \mathbb{R}$ and $l_t^h = 0$. Withdrawals are $w_t^h = 0$, and consumption is $c_t^h = \frac{M_t^h}{p_t}$;
3. $\varepsilon_t^h = \underline{\varepsilon} = 0$, $r_t^{b(h)} < 0$ and $l_t^h = +\infty$. Withdrawals are $w_t^h = D_t^h$, and consumption is $c_t^h = 0$ (run);
4. $\varepsilon_t^h = \underline{\varepsilon} = 0$, $\left(r_t^{b(h)}, l_t^h\right)$ such that $r_t^{b(h)} \geq 0$ and/or $l_t^h = 0$. Withdrawals are $w_t^h = 0$, and consumption is $c_t^h = 0$.

Next, I take as given the choices at night and I analyze the decisions during the day, in order to verify the guesses about M_t^h, D_t^h, and K_t^h, and about the value function. I use the distinctions among the four cases at night, taking into account the beliefs of households $\Pr_t^h$ about $r_t^{b(h)}$ and l_t^h, and the probability distribution over ε_t^h given by (1). The maximization problem during the day is:

$$\max_{\eta_t^M,\eta_t^D,\eta_t^K}\left\{\text{Pr}_t^h\left(r_t^{b(h)}\in\mathbb{R},l_t^h=+\infty\right)\kappa\left[\bar{\varepsilon}\log\left(\frac{\eta_t^M A_t^h+\eta_t^D A_t^h}{p_t}\right)+\frac{\beta}{1-\beta}\log\left(\eta_t^K A_t^h\left(1+R_t^K\right)\right)\right]+\right.$$
$$\text{Pr}_t^h\left(r_t^{b(h)}\in\mathbb{R},l_t^h=0\right)\kappa\left[\bar{\varepsilon}\log\left(\frac{\eta_t^M A_t^h}{p_t}\right)+\frac{\beta}{1-\beta}\log\left(\eta_t^K A_t^h\left(1+R_t^K\right)+\eta_t^D A_t^h\left(1+r_t^{b(h)}\right)\right)\right]+$$
$$\text{Pr}_t^h\left(r_t^{b(h)}<0,l_t^h=+\infty\right)(1-\kappa)\left[\frac{\beta}{1-\beta}\log\left(\eta_t^K A_t^h\left(1+R_t^K\right)+\eta_t^M A_t^h+\eta_t^D A_t^h\right)\right]+$$
$$\left.\left[1-\text{Pr}_t^h\left(r_t^{b(h)}<0,l_t^h=+\infty\right)\right](1-\kappa)\left[\frac{\beta}{1-\beta}\log\left(\eta_t^K A_t^h\left(1+R_t^K\right)+\eta_t^M A_t^h+\eta_t^D A_t^h\left(1+r_t^{b(h)}\right)\right)\right]\right\} \tag{60}$$

subject to the budget constraint (14) which can be written:

$$\eta_t^M A_t^h+\eta_t^D A_t^h+\eta_t^K A_t^h\leq A_t^h.$$

All terms A_t^h can be factored out of the logs in the objective function, implying:

$$\max_{\eta_t^M,\eta_t^D,\eta_t^K}\left\{\text{Pr}_t^h\left(r_t^{b(h)}\in\mathbb{R},l_t^h=+\infty\right)\kappa\left[\bar{\varepsilon}\log\left(\frac{\eta_t^M+\eta_t^D}{p_t}\right)+\frac{\beta}{1-\beta}\log\left(\eta_t^K\right)\right]+\right.$$
$$\text{Pr}_t^h\left(r_t^{b(h)}\in\mathbb{R},l_t^h=0\right)\kappa\left[\bar{\varepsilon}\log\left(\frac{\eta_t^M}{p_t}\right)+\frac{\beta}{1-\beta}\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^D\left(1+r_t^{b(h)}\right)\right)\right]+$$
$$\text{Pr}_t^h\left(r_t^{b(h)}<0,\,l_t^h=+\infty\right)(1-\kappa)\left[\frac{\beta}{1-\beta}\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^M+\eta_t^D\right)\right]+$$
$$\left.\left[1-\text{Pr}_t^h\left(r_t^{b(h)}<0,\,l_t^h+\infty\right)\right](1-\kappa)\left[\frac{\beta}{1-\beta}\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^M+\eta_t^D\left(1+r_t^{b(h)}\right)\right)\right]\right\} \tag{61}$$

and the budget constraint can be written (with equality) $\eta_t^M+\eta_t^D+\eta_t^K=1$. The objective function and the budget constraint are independent of A_t^h, and thus the optimal η_t^M, η_t^D, and η_t^K are independent of A_t^h too. The variables η_t^M, η_t^D and $\eta_t^K\in[0,1]$ to satisfy the non-negativity constraints on money, deposits, and capital. The guesses about the policy functions are thus verified. The optimal values of η_t^M, η_t^D, and η_t^K can be computed by taking first-order conditions of (61) and solving the resulting system of equations, subject to $\eta_t^M+\eta_t^D+\eta_t^K=1$ and η_t^M, η_t^D, and $\eta_t^K\in[0,1]$.

Finally, I verify the guess for the value function. To do so, I rewrite the Bellman equation (13) omitting terms that do not depend on A_t^h. Using equation (60) and the guess for $V_t\left(A_t^h\right)$:

$$\frac{1}{1-\beta}\log A_t^h=\kappa\bar{\varepsilon}\log\left(A_t^h\right)+\frac{\beta}{1-\beta}\log A_t^h$$

this expression holds true because $\kappa\bar{\varepsilon} = 1$ from equation (2), confirming the guess about the value function. □

Proof of Proposition 4.2. The variable $\hat{r}_t^b$ can be written:

$$1 + \hat{r}_t^b = \frac{K_t^b \left(Q_{t+1} + Zp_t\right)}{D_t^b - w_t^b} =$$

$$= \left(\frac{Q_{t+1} + Zp_t}{Q_t}\right) \frac{N_t^b + (1-\kappa) D_t^b}{D_t^b (1-\kappa)} = \left(\underbrace{\frac{Q^* + Zp_t}{Q_t}}_{=1+R_t^K}\right) \left(1 + \frac{N_t^b}{D_t^b (1-\kappa)}\right) \tag{62}$$

where the first equality uses the definition of $\hat{r}_t^b$ in equation (10); the second equality uses the results $K_t^b = \left(N_t^b + (1-\kappa) D_t^b\right)/Q_t$ and $w_t^b = M_t^b = \kappa D_t^b$ of Proposition 3.1; and the third equality rearranges and notes that the term in the first parentheses is the nominal return on capital, using equation (3).

Since prices are fixed by assumption (and thus $N_t^b < 0$ is fixed as well because it depends on state variables and on Q_t; see equation (4)) and $D_t^b \geq 0$ due to a non-negativity constraint,

$$\frac{\partial r_t^b}{\partial \left[\int_{\mathbb{H}(b)} D_t^h dh\right]} = \frac{\partial r_t^b}{\partial D_t^b} = \left(1 + R_t^K\right) \left(-\frac{N_t^b}{\left[D_t^b (1-\kappa)\right]^2}\right) > 0.$$

□

Proof of Proposition 5.1. By assumption, there exists an equilibrium with money supply $M_t = \overline{M}(1+\mu_t)$, in which the monetary injection $\mu_t \overline{M}$ is implemented with asset purchases. The 16 equations that characterize the equilibrium are described in Appendix D.4. Among them, there is:

1. the budget constraint of banks, holding with equality:

$$M_t^b + Q_t K_t^b = D_t^b + N_t^b; \tag{63}$$

2. the actual return on deposits paid by insolvent banks:

$$1 + r_t^b = \frac{K_t^b \left(Q^* + Zp_t\right)}{D_t^b - w_t^b}. \tag{64}$$

Let $L_t^b = L_t^{CB} = \mu_t \overline{M}$ and define $\tilde{K}_t^b = \frac{L_t^b}{Q_t}$. Then I can add L_t^b on both sides of equation (63) and, using the definition of $\tilde{K}_t^b$:

$$M_t^b + Q_t K_t^b + Q_t \tilde{K}_t^b = D_t^b + L_t^b + N_t^b.$$

Defining $\hat{K}_t^b = K_t^b + \tilde{K}_t^b$:

$$M_t^b + Q_t \hat{K}_t^b = D_t^b + L_t^b + N_t^b. \tag{65}$$

From Section 5.1.2, given prices $R_t^D = R_t^{CB} = R_t^K$, bank b chooses $M_t^b = \kappa D_t^b$; any D_t^b is a solution; any L_t^b is a solution; and $\hat{K}_t^b$ solves equation (65). Therefore, the choices D_t^b and M_t^b from the equilibrium with asset purchases, together with $\hat{K}_t^b = K_t^b + \tilde{K}_t^b$ and $L_t^b = L_t^{CB}$, are optimal for banks.

Next, adding and subtracting $L_t^b\left(1 + R_t^K\right)$ on the numerator on the right-hand side of equation (64), and using the definition of $\tilde{K}_t^b$:

$$\begin{aligned} 1 + r_t^b &= \frac{K_t^b\left(Q^* + Zp_t\right) + \tilde{K}_t^b Q_t\left(1 + R_t^K\right) - L_t^b\left(1 + R_t^K\right)}{D_t^b - w_t^b} \\ &= \frac{K_t^b\left(Q^* + Zp_t\right) + \tilde{K}_t^b Q_t\left(\frac{Q^* + Zp_t}{Q_t}\right) - L_t^b\left(1 + R_t^K\right)}{D_t^b - w_t^b} \\ &= \frac{\hat{K}_t^b\left(Q^* + Zp_t\right) - L_t^b\left(1 + R_t^K\right)}{D_t^b - w_t^b} \end{aligned} \tag{66}$$

where the second line uses equation (3) evaluated at $Q_t = Q^*$ (because the economy reverts to normal in $t+1$) and the last line uses $\hat{K}_t^b = K_t^b + \tilde{K}_t^b$. Comparing the last result with equation (31), r_t^b is also the actual return on deposits in an economy where banks get loans L_t^b that are senior to deposits.

Equations (65) and (66), together with the remaining 14 equations that described the original equilibrium, describe an equilibrium with loans to banks with the same prices and quantities but the amount of capital chosen by banks (K_t^b is replaced by $\hat{K}_t^b$). The choices of the representative household are unchanged because they depend only on prices and on the household's own state variable $\boldsymbol{X}_t^h$. The market clearing conditions for money, deposits, and consumption goods are also unchanged. The market clearing for capital (34) in the equilibrium with loans to banks must hold by Walras' Law. □

C Bankless crisis equilibrium

This appendix presents a bad equilibrium in which all banks are insolvent during the day of time t. The banking system is shut down for a period, and the economy reverts to normal in $t+1$. First, I analyze in more detail the liquidation of insolvent banks (as described by footnote 12) and the role of the non-negativity constraints on money and capital in the problem of bank (8). Second, Proposition C.1 states the conditions under which the bankless crisis equilibrium exists; in particular, since all banks are insolvent, the existence of this equilibrium does not require the shocks to capital $\underline{\psi}$ and $\overline{\psi}$ to hit the economy, and it does not hinge on asymmetric information as well. Third, I briefly discuss the sensitivity of the existence of this equilibrium to some timing assumptions; the bankless crisis equilibrium hinges on the assumption that it takes some time for new banks to enter the economy, but a rigorous analysis of entry in the banking market is outside the scope of this paper.

Liquidation of insolvent banks and non-negativity constraint on capital. Consider an active bank b with initial deposits $d_{t-1}^b > 0$ and negative net worth $N_t^b < 0$. The bank is able to operate if D_t^b is large enough and $N_t^b + D_t^b > 0$. In this case, the bank has resources to hold positive amounts of money and capital. If, on the contrary, the value of deposits D_t^b is small and $N_t^b + D_t^b < 0$, the non-negativity constraints on money and capital $M_t^b \geq 0$ and $K_t^b \geq 0$ become binding. Recall that d_{t-1}^b represents the pre-existing deposits at bank b; if many preexisting deposits are not rolled over during the day by depositors, the bank might not have enough resources to pay depositors. Footnote 12 states that, if this occurs, bank b is shut down in the day of time t (in the sense that it is forced to choose $M_t^b = 0$ and $K_t^b = 0$), and it is liquidated immediately (in the sense that preexisting deposits are repaid pro-rata using the value of the current assets of bank b, defined by $K_{t-1}^b \left(1 + \psi_t^b\right) Q_t + m_{t-1}^b$). If household h chooses deposits $D_t^h > 0$ at a bank $b(h)$ that is liquidated in the day of time t, then the household is depositing part of its wealth at a bank in which $M_t^{b(h)} = 0$ and $K_t^{b(h)} = 0$ and in which all the available resources are used to pay preexisting depositors. Thus, all the deposits D_t^h are lost (implying $r_t^{b(h)} = -1$) and cannot be withdrawn at night because the bank is forced to hold $M_t^{b(h)} = 0$; thus, $l_t^h = 0$.[42]

Existence of bankless crisis equilibrium. Proposition C.1 states the condition for the existence of the bankless crisis equilibrium and describes it. The proof is provided at the end of the Section.

Proposition C.1. *(Bankless equilibrium) If the state of the economy $\boldsymbol{X}_t$ satisfies:*

$$\boldsymbol{Pr}_t^B \left(\boldsymbol{X}_t^b \middle| K_{t-1}^b \left(1 + \psi_t^b\right) \left[1 - \frac{\kappa\left(1-\kappa\right)}{1 - \beta + 2\beta\kappa\left(1-\kappa\right) + \kappa^2}\right] \frac{\overline{M}}{\overline{K}} \frac{\beta}{1-\beta} + m_{t-1}^b - d_{t-1}^b < 0 \right) = 1$$

and:

$$K_{t-1}^s \left[1 - \frac{\kappa\left(1-\kappa\right)}{1 - \beta + 2\beta\kappa\left(1-\kappa\right) + \kappa^2}\right] \frac{\overline{M}}{\overline{K}} \frac{\beta}{1-\beta} + m_{t-1}^s - d_{t-1}^s < 0$$

for all $s \in \mathbb{S}$, then there exists an equilibrium such that:

[42] Alternatively, the results $r_t^{b(h)} = -1$ can be obtained as follows. Consider a bank b with $N_t^b < 0$, and assume that bank b issues $D_t^b = \delta$, where $\delta > 0$ but small, and then take the limit as $\delta \to 0$. Since δ is small, the non-negativity constraint on money and capital is still binding, so $M_t^b = 0$ and $K_t^b = 0$. Using equation (10), then $r_t^b \to -1$ as $\delta \to 0$.

- *prices are:*[43]

$$Q_t = \frac{\overline{M}}{\overline{K}} \frac{\beta}{1-\beta} \left[1 + \frac{\kappa (1-\kappa)}{1 - \beta + 2\beta\kappa (1-\kappa) + \kappa^2} \right],$$

$$p_t = \frac{\kappa \overline{M}}{Z \overline{K}},$$

$$1 + R_t^D = 1 + R_t^K = \frac{Q^* + Z p_t}{Q_t},$$

where Q^* is the steady-state price (see Definition A.3).

- *dividends paid by exiting banks are $\pi_t = 0$;*
- *all banks subject to exit shocks have negative net worth, $N_t^s < 0$ for all $s \in \mathbb{S}$;*
- *all active banks have negative net worth, $N_t^b < 0$ for all $b \in \mathbb{B}$ and choose $M_t^b = D_t^b = K_t^b = 0$;*
- *household $h \in \mathbb{H}$ has beliefs $Pr_t^h \left(r_t^{b(h)} = -1,\, l_t^h = 0 \right) = 1$; and its choice is given by Proposition 3.3, $\eta_t^D = 0$, $\eta_t^K = 1 - \eta_t^M$, and:*

$$\eta_t^M = \frac{(1-\beta)(1 - \beta + 2\beta\kappa + \kappa^2 - 2\beta\kappa^2)}{1 + \kappa^2 - \beta(1 - \kappa + \kappa^2)}, \tag{67}$$

- *actual return on deposits and limits on withdrawals are $r_t^b = -1$ for all b and $l_t^h = 0$ for all h.*

A critical assumption of Proposition C.1 is that banks that become insolvent at the beginning of time t are not replaced by new banks. If instead new banks enter the market to replace all banks liquidated at the beginning of time t, households would simply move their deposits from old failed banks to new banks. But, if this were the case, the price of capital would be Q^* as in the good equilibrium, and thus old banks would not fail in the first place.

Proof of Proposition C.1. Using (4) and the assumptions of the Proposition about the state of the economy, then the net worth of both active and exiting banks is negative. Given $R_t^D = R_t^K$, choices of banks follow from the non-negativity constraint on money and capital, from $N_t^b < 0$, and footnote 12. The actual return on deposits and the limits on withdrawals follow from the fact that no bank is active, and the non-negativity constraints on money and capital are binding as discussed above.

The choice $D_t^h = 0$ (and thus $\eta_t^D = 0$) follows trivially from households' beliefs. Thus, using equation (61) (from the proof of Proposition 3.3, see Appendix B), household beliefs (that trivially

[43] Restrictions on parameters can be added to make sure that Q_t is lower than the good-equilibrium price Q^*. Such restrictions can be verified to hold numerically, in the sense that I have not found any parameter values for which they are not satisfied.

satisfy rationality), and the constraint $\eta_t^M + \eta_t^D + \eta_t^K = 1$ (thus $\eta_t^K = 1 - \eta_t^M$):

$$\max_{\eta_t^M} \left\{ \kappa\bar{\varepsilon} \log\left(\frac{\eta_t^M}{p_t}\right) + \kappa \frac{\beta}{1-\beta} \log\left[\left(1-\eta_t^M\right)\left(1+R_t^K\right)\right] + \left[(1-\kappa)\frac{\beta}{1-\beta}\log\left(\left(1-\eta_t^M\right)\left(1+R_t^K\right)+\eta_t^M\right)\right]\right\}$$

subject to η_t^M, η_t^D, $\eta_t^K \in [0,1]$. The variable η_t^M solves the FOC:

$$\frac{1}{\eta_t^M} - \kappa\frac{\beta}{1-\beta}\frac{1}{1-\eta_t^M} - (1-\kappa)\frac{\beta}{1-\beta}\frac{R_t^K}{(1-\eta_t^M)(1+R_t^K)+\eta_t^M} = 0$$

and using the nominal interest rate (3), the price Q_t in the statement of the Proposition, and Q^* from definition A.3, equation (67) follows. Since $D_t^h = 0$, $w_t^h = 0$ from the constraint (15).

Dividends follow from equation (37) and the fact that $N_t^s < 0$ for all $s \in \mathbb{S}$.

The market clearing condition for money (38) requires $\int_{\mathbb{H}} M_t^h dh = \overline{M}$, which holds because the wealth of the representative household is $\overline{A}_t = \overline{K}Q_t + \overline{M}$ (banks hold no capital and no money, and $\pi_t = 0$) and $\eta_t^D \overline{A}_t = \overline{M}$ where the equality follows using (67). Market clearing for deposits also holds trivially. Market clearing for goods follows from the fact that only a fraction κ of households (those who are impatient) buy consumption goods at night (and therefore consumption expenditure is $\kappa\overline{M}$). The total demand for goods is thus $\frac{\kappa\overline{M}}{p_t} = Z\overline{K}$ (and it is thus equal to the supply) where the equality follows using p_t in the statement of the Proposition. The market clearing for capital holds by Walras' Law.

Finally, I need to show that $Q_{t+1} = Q^*$ in order to show that R_t^K satisfies (3). This is the case because, in $t+1$, the state variables of active banks are $\boldsymbol{X}_{t+1}^b = \{(0,0,0), 0\}$ (because new banks with no previous assets and liabilities enter the market). Thus the conditions of Proposition A.7 are satisfied. □

D Bad equilibrium

This appendix provides more details on how I solve for the bad equilibria. I first describe households' beliefs, then present the full set of equations that is used to solve for the equilibrium, and then I describe the solution method. Most of the appendix focuses on the model with no policy intervention, but I provide a brief extension to the case with monetary injections at the end.

D.1 Households' beliefs

Consider the state variables of banks $\left\{\boldsymbol{X}_t^b\right\}_{b\in[0,1]}$ described in Proposition A.6. A fraction α of banks is hit by a negative shock $\underline{\psi}$, and the remaining fraction $1-\alpha$ is hit by $\overline{\psi}$. I conjecture that, at night, households run on banks hit by $\underline{\psi}$, and then I solve for the "candidate equilibrium" using the non-linear system of equations described below. The conjecture is verified if $r_t\left(\underline{\psi}\right) < 0$ so that "running" is indeed the optimal choice of households (see Proposition 3.3) and the "candidate equilibrium" is an equilibrium. The initial conjecture is confirmed for a wide range of parameters. For some values of the parameters, $0 < r_t\left(\underline{\psi}\right) < R_t^D$ so there exists no bad equilibria with runs on a fraction of the banking system for those parameters.

To describe households beliefs during the day, note that household $h \in \mathbb{H}$ holding deposits at bank $b\left(h\right)$ faces one of the following possibilities at night.

1. With probability $1-\alpha$, bank $b\left(h\right)$ pays the promised return $R_t^D > 0$. Therefore, bank $b\left(h\right)$ is not subject to runs, and household h can withdraw any amount $\leq D_t^h$.
2. With probability α, bank $b\left(h\right)$ is bankrupt in $t+1$, paying a return $r_t\left(\underline{\psi}\right) < 0$. Therefore the optimal choice for household h is to run to try to withdraw as much as possible:
   (a) with probability $f_t^{b(h)}$, household h is "first in line" ($l_t^h = +\infty$) so it is able to withdraw any amount of money $w_t^h \leq D_t^h$;
   (b) with probability $1-f_t^{b(h)}$, household h is "last in line" ($l_t^h = 0$), so it is unable to withdraw money, $w^h\left(n_t^h\middle|\, l_t^h = 0\right) = 0$. In this case, if household h is impatient ($\varepsilon_t = \bar{\varepsilon}$), it is able to buy some consumption goods only if it chose to hold some money $M_t^h > 0$ during the day.

Therefore, the rational beliefs of households are:

$$\text{Pr}_t^h\left(r_t^{b(h)} = R_t^D,\, l_t^h = +\infty\right) = 1-\alpha \tag{68}$$

$$\text{Pr}_t^h\left(r_t^{b(h)} = r_t\left(\underline{\psi}\right) < 0\,, l_t^h = +\infty\right) = f_t^{b(h)}\alpha \tag{69}$$

$$\text{Pr}_t^h\left(r_t^{b(h)} = r_t\left(\underline{\psi}\right) < 0\,, l_t^h = 0\right) = \left(1-f_t^{b(h)}\right)\alpha. \tag{70}$$

D.2 Equations describing the equilibrium

I now describe the full set of equations that I use to solve for the equilibrium. I organize the discussion in three building blocks: households, banks, and market clearing. The equations refer to the general model with dividends and bankers presented in Appendix A. The equations that refer to the model presented in the main text, Section 2, are a special case that can be obtained by setting $\lambda = 0$.

Households. Households hold capital and money that is not held by banks, plus deposits. The total wealth of households $\overline{A}_t$ is determined by an expression similar to equation (48) (total value of capital and money in the economy minus what is held by banks, plus value of deposits):

$$\overline{A}_t = \left[\overline{K} - K^* - \lambda \frac{K^*}{1-\lambda}\right] Q_t + \left(\overline{M} - m^* - \lambda \frac{m^*}{1-\lambda}\right) + \\ + \lambda\alpha \left[\frac{K^*}{1-\lambda}\left(1+\underline{\psi}\right) Q_t + \frac{m^*}{1-\lambda}\right] + (1-\lambda\alpha)\, d^*. \quad (71)$$

The first term is the value of capital: the total supply $\overline{K}$ minus capital owned by active banks K^* (defined by equation 40) minus capital owned by banks under liquidation $\lambda\frac{K^*}{1-\lambda}$. The second term is the value of money; similarly to capital, it is given by the total supply of money $\overline{M}$ minus the money owned by active banks m^* (defined by equation 41) minus the money held by banks under liquidation $\lambda\frac{m^*}{1-\lambda}$. The third term is the value of deposits from banks hit by the exit shock and by $\underline{\psi}$ (such banks are insolvent, so all their assets are used to repay depositors). The last term is the value of deposits at other banks.

The household problem (13) can be rewritten:[44]

$$\max_{\eta_t^M,\eta_t^D,\eta_t^K} \Bigg\{ (1-\alpha)\left[\kappa\bar{\varepsilon}\log\left(\eta_t^M+\eta_t^D\right) + \frac{\beta}{1-\beta}\kappa\log\left(\eta_t^K\right) \right. \\ \left. + \frac{\beta}{1-\beta}(1-\kappa)\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^D\left(1+R_t^D\right)+\eta_t^M\right)\right] \\ +\alpha f_t^b\left[\kappa\bar{\varepsilon}\log\left(\eta_t^M+\eta_t^D\right)+\frac{\beta}{1-\beta}\Big(\kappa\log\eta_t^K+(1-\kappa)\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^D+\eta_t^M\right)\Big)\right] \\ +\alpha\left(1-f_t^b\right)\left[\kappa\bar{\varepsilon}\log\eta_t^M+\frac{\beta}{1-\beta}\kappa\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^D\left(1+r_t^{b(h)}\right)\right)\right. \\ \left.+\frac{\beta}{1-\beta}(1-\kappa)\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^D\left(1+r_t^{b(h)}\right)+\eta_t^M\right)\right]\Bigg\}$$

subject to $\eta_t^M+\eta_t^D+\eta_t^K \leq 1$. Using:

$$\eta_t^K = 1-\eta_t^M-\eta_t^D, \quad (72)$$

[44] I use the derivation in the proof of Proposition 3.3, in Appendix B, in particular equation (61), combined with the beliefs specified by equations (68), (69), and (70). The last line of equation (61) encompasses the cases $\left(\varepsilon_t^h, r_t^{b(h)}, l_t^h\right) \in \left\{\left(0, R_t^D, +\infty\right), \left(0, r_t\left(\underline{\psi}\right), 0\right)\right\}$.

the FOCs with respect to η_t^M and η_t^D are:

$$(1-\alpha)\left[\frac{1}{\eta_t^D+\eta_t^M}-\frac{\beta}{1-\beta}\left(\frac{\kappa}{1-\eta_t^M-\eta_t^D}+\frac{(1-\kappa)\,R_t^K}{1+(1-\eta_t^M)\,R_t^K+\eta_t^D\,(R_t^D-R_t^K)}\right)\right]$$
$$+\alpha f_t^b\left[\frac{1}{\eta_t^D+\eta_t^M}-\frac{\beta}{1-\beta}\kappa\frac{1}{1-\eta_t^M-\eta_t^D}-\frac{\beta\,(1-\kappa)}{1-\beta}\frac{R_t^K}{1+(1-\eta_t^M-\eta_t^D)\,R_t^K}\right]$$
$$+\alpha\left(1-f_t^b\right)\left[\frac{1}{\eta_t^M}-\frac{\beta}{1-\beta}\kappa\frac{1+R_t^K}{(1-\eta_t^M)\,(1+R_t)+\eta_t^D\left(r_t^{b(h)}-R_t^K\right)}\right.$$
$$\left.-\frac{\beta\,(1-\kappa)}{1-\beta}\frac{R_t^K}{1+(1-\eta_t^M)\,R_t^K+\eta_t^D\left(r_t^{b(h)}-R_t^K\right)}\right]=0 \quad (73)$$

$$(1-\alpha)\left[\frac{1}{\eta_t^D+\eta_t^M}-\frac{\beta}{1-\beta}\left(\frac{\kappa}{1-\eta_t^M-\eta_t^D}-\frac{(1-\kappa)\left(R_t^D-R_t^K\right)}{1+(1-\eta_t^M)\,R_t^K+\eta_t^D\,(R_t^D-R_t^K)}\right)\right]$$
$$+\alpha f_t^b\left[\frac{1}{\eta_t^D+\eta_t^M}-\frac{\beta}{1-\beta}\kappa\frac{1}{1-\eta_t^M-\eta_t^D}-\frac{\beta\,(1-\kappa)}{1-\beta}\frac{R_t^K}{1+(1-\eta_t^M-\eta_t^D)\,R_t^K}\right]$$
$$+\alpha\left(1-f_t^b\right)\left[\frac{\beta}{1-\beta}\kappa\frac{r_t^{b(h)}-R_t^K}{(1-\eta_t^M)\,(1+R_t^K)+\eta_t^D\left(r_t^{b(h)}-R_t^K\right)}\right.$$
$$\left.+\frac{\beta\,(1-\kappa)}{1-\beta}\frac{r_t^{b(h)}-R_t^K}{1+(1-\eta_t^M)\,R_t^K+\eta_t^D\left(r_t^{b(h)}-R_t^K\right)}\right]=0 \quad (74)$$

I numerically verify that η_t^M, η_t^D, and η_t^K lie in the interval $[0,1]$. The behavior of households is thus described by equations (71), (72), (73), and (74) that determine, given prices, $\overline{A}_t$, η_t^K η_t^M and η_t^D.

Banks. The net worth of solvent banks $N_t\left(\overline{\psi}\right)$ and insolvent banks $N_t\left(\underline{\psi}\right)$ is computed using equation (4) evaluated at the respective values of ψ_t^b and using the values of capital K^*, money m^*, and deposits d^* from equations (40), (41), and (42). I then use the budget constraint of banks (6) separately for solvent banks (banks hit by $\overline{\psi}$) and insolvent banks (banks hit by $\underline{\psi}$), evaluated at the optimal choice of money described by Proposition 3.1 and taking as given the demand of deposits by households (using $R_t^K=R_t^D$, banks are indifferent about the amount of deposits). The behavior of banks is thus described by four equations (the two definitions of net worth and the two budget constraints) that determine, given prices, $N_t\left(\overline{\psi}\right)$, $N_t\left(\underline{\psi}\right)$, $K_t\left(\overline{\psi}\right)$ and $K_t\left(\underline{\psi}\right)$.

The actual return on deposits of insolvent banks, $r_t\left(\underline{\psi}\right)$ is given by equation (10), using $Q_{t+1}=Q^*$ and $w_t^b=M_t^b=\kappa D_t^b$. The fraction of depositors served during a run is given by (11) evaluated at $D_t^b = \eta_t^D \overline{A}_t$ and $M_t^b = \kappa D_t^b = \kappa \eta_t^D \overline{A}_t$ that imply $f_t = \kappa$. Therefore I have two equations that determine $r_t\left(\underline{\psi}\right)$ and f_t.

To pin down dividends π_t, I use the state variable of banks hit by the exit shock, described in Proposition A.6, the fact that there is a mass λ of such banks, and the expression for dividends in equation (37):

$$\pi_t = \lambda\left(1-\alpha\right)\frac{K^*\left(1+\overline{\psi}\right)Q_t + m^* - d^*}{1-\lambda} = \lambda\left(1-\alpha\right)\frac{N_t\left(\overline{\psi}\right)}{1-\lambda}.$$

The equalities follows from the fact that a fraction α of banks is hit by $\underline{\psi}$ and has negative net worth; thus, all the assets of such insolvent banks are used to repay depositors, and banks hit by $\underline{\psi}$ do not contribute to dividends.

Market clearing. Withdrawal and consumption decisions of households (Proposition 3.3) together with the market clearing condition for goods, equation (39), imply:

$$Z\overline{K}p_t = \pi_t + \kappa\left[\left(1-\alpha\right)\left(M_t^h + D_t^h\right) + \alpha f_t\left(M_t^h + D_t^h\right) + \alpha\left(1-f_t\right)M_t^h\right] \tag{75}$$

where $M_t^h = \eta_t^M A_t^h$, $D_t^h = \eta_t^D A_t^h$, and I consider the wealth $A_t^h = \overline{A}_t$ of the representative household. Equation (75) says that the total consumption expenditure is equal to dividends (bankers spend all their dividends π_t to buy consumption) plus the consumption expenditure of the fraction κ of households that are impatient. A fraction $1-\alpha$ of impatient households deal with solvent banks and are able to withdraw D_t^h, so their consumption expenditure is $M_t^h + D_t^h$. A fraction αf_t of impatient households are first in line during runs, so they can withdraw and they spend $M_t^h + D_t^h$ as well. A fraction $\alpha\left(1-f_t\right)$ are last in line during a run and consume using only money M_t^h. The market clearing condition for money during the day is given by equation (38) where $\int M_t^b db = \kappa \int D_t^b db = \kappa \eta_t^D \overline{A}_t$ and $\int M_t^h dh = \eta_t^M \overline{A}_t$.

The two market clearing conditions for goods and money determine the price level p_t and the price of capital Q_t. The return on capital R_t^K is then determined by (3), evaluated at $Q_{t+1} = Q^*$ (see Proposition A.7 and the discussion thereafter).

The return on deposits is $R_t^D = R_t^K$ in order to satisfy the market clearing condition for deposits.

D.3 Solution method

I obtain a non-linear system of 15 polynomial equations with 15 unknowns. Since I take as given the state of the economy $\boldsymbol{X}_t$ and the price of capital $Q_{t+1} = Q^*$ in $t+1$, I am just solving a static problem, for period t. I solve the system in Mathematica using the command NSolve and selecting the real solutions that satisfy all the non-negativity constraints on money, deposits, and capital imposed on the maximization problem of households and banks, and the constraints η_t^M, η_t^D, $\eta_t^K \in [0, 1]$. I use the values of parameters shown in Table **??** and the initial conditions for banks in equations (40), (41), and (42).

The command NSolve in Mathematica computes the numerical Gröbner bases associated with the system of polynomial equations and then uses eigensystem methods to extract numerical roots, finding all solutions to the system. See Kubler and Schmedders (2010) for an introduction to Gröbner bases applied to the computation of equilibria in economic models, and Lichtblau (2000) for a detailed description of the solution method of NSolve.

D.4 Monetary policy analysis: numerical solution

I use the same approach as in the baseline model, modifying the 15 equations listed there as follow.

- The budget constraint of banks (6) is replaced by (29).
- The actual return on deposits of insolvent banks (10) is replaced by (31) or by (32) depending on the policy.
- The market clearing condition for money market is replaced by the one in (34). Alternatively, using the model with bankers in Appendix A, the market clearing condition for money is $\int_{\mathbb{B}} M_t^b db + \int_{\mathbb{H}} M_t^h dh = \overline{M}(1 + \mu_t) - \pi_t$.
- The set of equations that must hold in equilibrium include (28), that determines the value of transfers T_{t+1}.